# 1. Interest Rate Derivatives (FRA, IRS, Interest Rate Futures and Options

## 1.1 Instruments

**Interest Rate Swap (IRS)** is an agreement between two parties to exchange cash flows based on a specified amount of principal for a set length of time. IRS is a long term agreement. In Poland maturities reach 10 years (in the world up to 30 years).

**FRA (forward rate agreement)** is a transaction in which two counterparties agree to a single exchange of cash flows based on fixed and a floating rate. A 3x9 FRA means a contract on a six-month WIBOR (in Poland) reference rate, three months forward. No payment will take place until floating rate six-month WIBOR is revealed after three months. In Poland settlement will take place in advance, eg. after 3 months. The share of the five most active banks in turnover accounts for almost 90 per cent. Most of transactions are speculative.

**Interest rate futures** contracts are traded on organized exchanges. In the world interest rate futures (eurodollar, T-bills, T-note, T-bond, municipal bond) contracts represent more than one-half of the entire futures market. In Poland there is still very small interest in interest rate futures. The first IR futures on WIBOR in Poland were introduced by Warsaw Commodities Exchange in February 1999 and after short period these contracts disappeared.

**IR options** are instruments which give the right to buy or sell interest rate sensitive instruments at a pre-determined interest rate. The price of the option is a premium paid on the second day after transaction. Options on forward interest rate (WIBOR) have been traded (OTC market) from 1998. PLN options are also written by London investment banks. Banks quote volatility.

### 1.1.1 IRS

In an interest rate swap (IRS) cash flows are denominated in the same currency. An interest swap does not involve an exchange of principal. The two parties arrange to make periodic exchanges of cash flows based on a common national principal and two separate interest rates, one that remains fixed and one that is reset (floating rate). All settlements are made on a net basis.

#### Terms of the swap transaction

1. notional principal,
2. floating rate and reference dates,
3. fixed rate,
4. settlement dates,
5. collateral,
6. currencies.

#### Long and Short Position

The two parties to the transaction are referred to as the pay-fixed and receive-fixed.

The fixed-rate payer is said to have “bought” the swap or taken the long position. The fixed-rate receiver is said to have ”sold”, or taken the short position. The floating rate is treated as the “commodity” that the two parties are trading.

## Payments

Fixed-rate payments are usually calculated using “actual/365” convention. The day-count convention for the swap floating rate-payments is usually “actual/360”. Under these conditions the formulas for the settlement calculations are following:

(1) $$OS_t = s * \frac{p_t}{365} * K$$

(2) $$OZ_t = LIBOR_{t-1} * \frac{p_t}{360} * K$$

where
$OS_t$ - fixed rate payment,
$OZ_t$ - floating-rate payment,
s - swap fixed rate, SFR,
$LIBOR_{t-1}$ – floating rate LIBOR at date t-1,
$p_t$ – number of days in period t,
t - period,
K - notional principal, NP.

**Problem 1. IRS**

A dealer quates the following swap fixed rates

| Maturity | Treasury Yield | Bid Swap Spread (bps) | Ask Swap Spread (bps) | Effective Fixed Swap Rate |
|---|---|---|---|---|
| 2 | 9,76% | 26 | 29 | 10,02% - 10,05% |
| 3 | 9,92% | 28 | 31 | 10,20% - 10,23% |
| 4 | 10,10% | 30 | 33 | 10,40% - 10,43% |
| 5 | 10,41% | 27 | 31 | 10,68% - 10,72% |
| 7 | 10,64% | 36 | 40 | 11,00% - 11,04% |
| 10 | 10,82% | 38 | 42 | 11,20% - 11,24% |
| 15 | 11,09% | 58 | 63 | 11,67% - 11,72% |

The dealer arranges IRS with a counterparty X with the following terms

| | |
|---|---|
| Origination date: | 2000-01-15 |
| Maturity date: | 2005-01-15 |
| Notional principal: | 10 000 000 |
| Fixed-rate payer | Counterparty X |
| Swap fixed rate: | 10,72% |
| Convention: | 365 |
| Fixed-rate receiver: | Dealer |
| Floating rate: | 6M WIBOR |
| Convention: | 360 |
| Settlement dates: | January 15th and July 15th of each year |
| WIBOR determination: | Determined in advance, paid in arrears |

The dealer arranges IRS with a counterparty Y with the following terms

| | |
|---|---|
| Origination date: | 2000-01-15 |
| Maturity date: | 2005-01-15 |
| Notional principal: | 10 000 000 |
| Fixed-rate payer | Dealer |
| Swap fixed rate: | 10,68% |
| Convention: | 365 |
| Fixed-rate receiver: | Counterparty Y |
| Floating rate: | 6M WIBOR |
| Convention: | 360 |
| Settlement dates: | January 15th and July 15th of each year |
| WIBOR determination: | Determined in advance, paid in arrears |

Required

(a) Draw a box-and-arrow diagram from a dealer's point of view.

(b) Calculate the swap cash flows assuming the 6M WIBOR path shown in the following table.

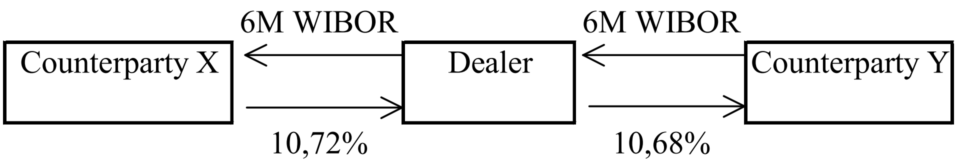
| Settlement Date | WIBOR |
|---|---|
| 2000-01-15 | 10,400% |
| 2000-07-15 | 10,600% |
| 2001-01-15 | 9,100% |
| 2001-07-15 | 8,100% |
| 2002-01-15 | 7,600% |
| 2002-07-15 | 9,100% |
| 2003-01-15 | 10,100% |
| 2003-07-15 | 10,500% |
| 2004-01-15 | 10,600% |
| 2004-07-15 | 8,400% |
| 2005-01-15 | 10,200% |

(c) Calculate the swap dealer's profit on each settlement date.

Calculate the swap dealer's profit assuming 90/360 conventions is used

**Solution**

(a)




(b)

| Settlement Date | Number of Days | 6M WIBOR | Floating Rate Receipt | Fixed Rate Payment | Counterparty X Net Receipt (Payment) |
|---|---|---|---|---|---|
| 2000-01-15 | - | 10,400% | - | - | - |
| 2000-07-15 | 182,00 | 10,600% | 525 778 | 534 532 | -8 754 |
| 2001-01-15 | 184,00 | 9,100% | 541 778 | 540 405 | 1 372 |
| 2001-07-15 | 181,00 | 8,100% | 457 528 | 531 595 | -74 067 |
| 2002-01-15 | 184,00 | 7,600% | 414 000 | 540 405 | -126 405 |
| 2002-07-15 | 181,00 | 9,100% | 382 111 | 531 595 | -149 483 |
| 2003-01-15 | 184,00 | 10,100% | 465 111 | 540 405 | -75 294 |
| 2003-07-15 | 181,00 | 10,500% | 507 806 | 531 595 | -23 789 |
| 2004-01-15 | 184,00 | 10,600% | 536 667 | 540 405 | -3 739 |
| 2004-07-15 | 182,00 | 8,400% | 535 889 | 534 532 | 1 357 |
| 2005-01-15 | 184,00 | 10,200% | 429 333 | 540 405 | -111 072 |
| | | | 4 796 000 | 5 365 874 | -569 874 |

| Settlement Date | Number of Days | 6M WIBOR | Fixed Rate Receipt | Floating Rate Payment | Counterparty Y Net Receipt (Payment) |
|---|---|---|---|---|---|
| 2000-01-15 | - | 10,400% | - | - | - |
| 2000-07-15 | 182,00 | 10,600% | 532 537 | 525 778 | 6 759 |
| 2001-01-15 | 184,00 | 9,100% | 538 389 | 541 778 | -3 389 |
| 2001-07-15 | 181,00 | 8,100% | 529 611 | 457 528 | 72 083 |
| 2002-01-15 | 184,00 | 7,600% | 538 389 | 414 000 | 124 389 |
| 2002-07-15 | 181,00 | 9,100% | 529 611 | 382 111 | 147 500 |
| 2003-01-15 | 184,00 | 10,100% | 538 389 | 465 111 | 73 278 |
| 2003-07-15 | 181,00 | 10,500% | 529 611 | 507 806 | 21 805 |
| 2004-01-15 | 184,00 | 10,600% | 538 389 | 536 667 | 1 722 |
| 2004-07-15 | 182,00 | 8,400% | 532 537 | 535 889 | -3 352 |
| 2005-01-15 | 184,00 | 10,200% | 538 389 | 429 333 | 109 056 |
| | | | 5 345 852 | 4 796 000 | 549 852 |

(c)
The dealer's cash flows are as follows

| Settlement Date | Receipt | Payment | Net Receipt |
|---|---|---|---|
| 2000-01-15 | - | - | - |
| 2000-07-15 | 534 532 | 532 537 | 1 995 |
| 2001-01-15 | 540 405 | 538 389 | 2 016 |
| 2001-07-15 | 531 595 | 529 611 | 1 984 |
| 2002-01-15 | 540 405 | 538 389 | 2 016 |
| 2002-07-15 | 531 595 | 529 611 | 1 984 |
| 2003-01-15 | 540 405 | 538 389 | 2 016 |
| 2003-07-15 | 531 595 | 529 611 | 1 984 |
| 2004-01-15 | 540 405 | 538 389 | 2 016 |
| 2004-07-15 | 534 532 | 532 537 | 1 995 |
| 2005-01-15 | 540 405 | 538 389 | 2 016 |
| | 5 365 874 | 5 345 852 | 20 022 |

If the 180/360 convention was used for both interest rates, the dealer's profit would be fixed and equal to
(10,72%-10,68%) * 180/360 * 10 000 000 = 2 000.

### 1.1.2 FRA

**Problem 2. FRA**

The actual bid-offer quates on FRA are following

| Period | Bid | Offer |
|---|---|---|
| 3 x 6 | 4,93% | 4,97% |
| 6 x 9 | 4,84% | 4,88% |
| 9 x 12 | 4,86% | 4,90% |
| 12 x 15 | 5,07% | 5,11% |
| 15 x 18 | 5,03% | 5,07% |
| 18 x 21 | 5,05% | 5,09% |
| 21 x 24 | 5,11% | 5,15% |

Required

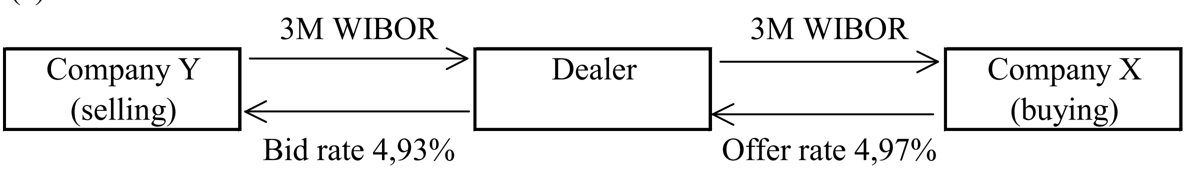
(a) Draw a box-and-arrow diagram for a 3x6 FRA.

(b) Suppose that the dealer buys 3x6 FRA from Company Y. Notional principal is $1 000 000. The 3M WiBOR is 5,0% on the rate determination date. Calculate the net payment.

(c) Calculate the net in advance payment.

**Solution**

(a)




(b)

The net payment in month 6 woould be (5,00%-Bid rate 4,93%) / 4 * 1 000 000 =185,2.

(c)

If settled in advance, the payment in month 3 would be
185,2 / (1+5,00% / 4 ) = 182,9.

## 1.2 Pricing Interest Rate. Up-front fee. Valuation (MTM)

A plain vanilla swap's price is its fixed rate (FSR – fixed swap rate). Swap pricing is not the same as swap valuation.

A plain vanilla swap pricing is the process of setting the fixed rate, so that the initial value of the swap is zero for both counterparties. Although the swap fixed rate is quoted off the Treasury yield curve, it is priced off an appropriate forward curve corresponding to the floating reference rate on the swap.

The swap's fixed rate should be established at the level so that the present value of the fixed cash flows eventually adjusted by an up-front fee equals the present value of the floating cash flows implied by the forward rates. These forward rates may be observed, calculated or estimated. The appropriate spot rates are used as discounting rates.

The process of valuation is called "mark-to-market". Thereafter it is positive for one counterparty making it an asset and negative for another counterparty making it a liability. The value of the swap is found by comparing FSR to the current swap fixed rate on a swap having the same terms and remaining time to maturity as the original transaction.

## 1.2.1 Swap with a LIBOR/WIBOR as a reference rate

### The FRA interest rates

In the United States the FRA prices are derived from the observed LIBOR forward curve (Eurodollar futures) because it indicates directly the levels of the floating interest rate that can be locked in by arbitrage transactions. In Poland instead WIBOR futures do no exist and the FRA prices may be derived only using implied forward rates, which are inferred from the WIBOR and WIBID spot interest rates and eventually interest rates observed in the FX swap market.

### Swap with a tenor up to 2 years

In the United States and in Poland the fixed rate of the interest rate swap with a tenor of up to two years is established using the observed FRA interest rates.

### Swap with a tenor between 2 and 10 years

In the United States the fixed rate of the interest rate swap with a tenor of between 2 and 10 years is established using the observed LIBOR forward curve (Eurodollar futures). In Poland WIBOR futures do not exist. The forward curve for WIBOR must be estimated off the forward curve for Treasury securities. The following procedure for a U.S. swap exceeding 10 years applies.

### Swap with a tenor exceeding 10 years

In the United States the fixed rate of the interest rate swap with a tenor exceeding 10 years is established using estimated forward rates. These estimated LIBOR forward rates are predicted using the forward rates for Treasury securities and independently forecasted TED (Treasury Eurodollar Difference). The forward rates for Treasury securities are implied forward rates calculated using spot Treasury curve. The spot Treasury curve is sometimes directly observed but it is usually derived using the "bootstrapping" procedure.

## 1.2.2 Swap with a Treasury yield as a reference rate

Assuming that floating-rate payments are made on the basis of a/360 and fixed-rate payments are made on the basis of a/365 (other assumptions will be introduced subsequently)

$$\sum_{t=1}^{T}\frac{{}_{t}f_{t\text{-}1}\frac{p_t}{360}K_t}{\left(1+z_t\frac{p_t}{360}\right)^t}=\sum_{t=1}^{T}\frac{s_T\frac{p_t}{365}K_t}{\left(1+z_t\frac{p_t}{360}\right)^t}+PP_0$$

where

${}_{t}f_{t\text{-}1}$ - forward rate,

$p_t$ – number of days in period t

$K_t$ – notional principal in period t,

$s_T$ – swap fixed rate,

$z_t$ – spot rate (discounting rate,

$PP_0$ – up-front fee,

t – period (t=1,2,...,T),

(3) $$PP_0=\sum_{t=1}^{T}\frac{{}_{t}f_{t\text{-}1}\frac{p_t}{360}K_t}{\left(1+z_t\frac{p_t}{360}\right)^t}-\sum_{t=1}^{T}\frac{s_T\frac{p_t}{365}K_t}{\left(1+z_t\frac{p_t}{360}\right)^t}$$

Swap fixed rate is equal to:

$$s_T=\frac{\sum_{t=1}^{T}\frac{{}_{t}f_{t\text{-}1}\frac{p_t}{360}}{\left(1+z_t\frac{p_t}{360}\right)^t}K_t}{\sum_{t=1}^{T}\frac{\frac{p_t}{365}}{\left(1+z_t\frac{p_t}{360}\right)^t}K_t}$$

## 1.2.3 Swap with a WIBOR (LIBOR) rate

(4) $$\sum_{t=1}^{T}\frac{{}_{t}f_{t\text{-}1}\frac{d_t-d_{t-1}}{360}K_t}{1+z_t\frac{d_t}{360}}=\sum_{t=1}^{T}\frac{s_T\frac{d_t-d_{t-1}}{365}K_t}{1+z_t\frac{d_t}{360}}$$

Swap fixed rate is equal to:

$$s_T=\frac{\sum_{t=1}^{T}\frac{{}_{t}f_{t\text{-}1}\frac{d_t-d_{t\text{-}1}}{360}}{1+z_t\frac{d_t}{360}}K_t}{\sum_{t=1}^{T}\frac{\frac{d_t-d_{t\text{-}1}}{365}}{1+z_t\frac{d_t}{360}}K_t}$$

## Problem 3. Pricing FRA rates using Eurodollar futures

Suppose today is: 20-11-98. Three-month LIBOR is 5,25%
The three-month forward rates for Eurodollar futures contracts are following:

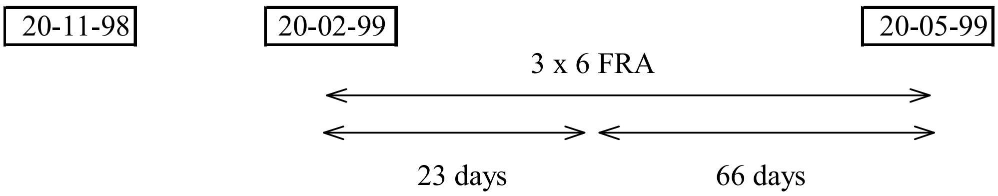
| Maturity | Rate |
|---|---|
| 12-98 | 5,19% |
| 03-99 | 4,86% |
| 06-99 | 4,86% |
| 09-99 | 4,89% |
| 12-99 | 5,16% |
| 03-00 | 5,02% |
| 06-00 | 5,09% |
| 09-00 | 5,15% |

Required
(a) Calculate the prices for a series 3 x 6, 6 x 9, 9 x 12, 12 x 15, 15 x 18, 18 x 21, 21 x 24 of FRA transactions based on Eurodollar futures.
(b) Calculate the prices for a series 6 x 12, 12 x 18, 18 x 24 of FRA using rates calculated in part (a).

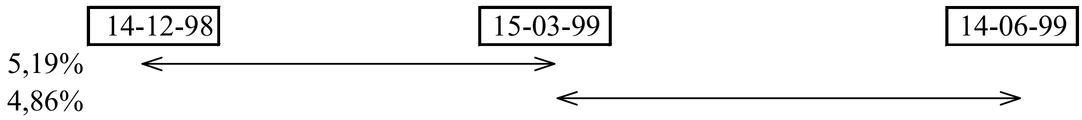
**Solution**

(a)

A 3 x 6 FRA rate is calculated as a weighted average of December and March Eurodollar futures rates. The number of days before and after March futures contract divided by the number of days for a FRA transaction are used as weights.




Eurodollar futures contracts




The FRA rates for a series of seven contracts:

| FRA | Method of calculation |
|---|---|
| 3 x 6 | (5,19% * 23 days + 4,86% * 66 days) : 89 days = 4,95% |
| 6 x 9 | (4,86% * 25 days + 4,86% * 67 days) : 92 days = 4,86% |
| 9 x 12 | (4,86% * 24 days + 4,89% * 68 days) : 92 days = 4,88% |
| 12 x 15 | (4,89% * 23 days + 5,16% * 69 days) : 92 days = 5,09% |
| 15 x 18 | (5,16% * 22 days + 5,02% * 68 days) : 90 days = 5,05% |
| 18 x 21 | (5,02% * 30 days + 5,09% * 62 days) : 92 days = 5,07% |
| 21 x 24 | (5,09% * 29 days + 5,15% * 63 days) : 92 days = 5,13% |

Calculations are made using the following table

| Futures maturity | Futures rate | FRA 1st date | Futures maturity | FRA 2nd date | Days ($p_1$) | Days ($p_2$) | Forward rate |
|---|---|---|---|---|---|---|---|
| 12-98 | 5,19% | 20-11-98 | 14-12-98 | 20-02-99 | 24 | 68 | 5,25% |
| 03-99 | 4,86% | 20-02-99 | 15-03-99 | 20-05-99 | 23 | 66 | 4,95% |
| 06-99 | 4,86% | 20-05-99 | 14-06-99 | 20-08-99 | 25 | 67 | 4,86% |
| 09-99 | 4,89% | 20-08-99 | 13-09-99 | 20-11-99 | 24 | 68 | 4,88% |
| 12-99 | 5,16% | 20-11-99 | 13-12-99 | 20-02-00 | 23 | 69 | 5,09% |
| 03-00 | 5,02% | 20-02-00 | 13-03-00 | 20-05-00 | 22 | 68 | 5,05% |
| 06-00 | 5,09% | 20-05-00 | 19-06-00 | 20-08-00 | 30 | 62 | 5,07% |
| 09-00 | 5,15% | 20-08-00 | 18-09-00 | 20-11-00 | 29 | 63 | 5,13% |

(b)

| FRA | Method of calculation |
|---|---|
| 6 x 12 | (1 + 4,86% * 92/360 ) (1 + 4,88% * 92/360 ) = (1 + FRA 6x12 * 184/360)<br>Thus FRA 6x12 = 4,90%. |
| 12 x 18 | (1 + 5,09% * 92/360 ) (1 + 5,05% * 90/360 ) = (1 + FRA 12x18 * 182/360)<br>Thus FRA 12x18 = 5,10%. |
| 18 x 24 | (1 + 5,07% * 92/360 ) (1 + 5,13% * 92/360 ) = (1 + FRA18 x24 * 184/360)<br>Thus FRA 18x24 = 5,13%. |

**Problem 4. Pricing Interest Rate Swap Off the FRA Curve**

The current FRA term structure is

| FRA | Rate | Days ($p_t$) |
|---|---|---|
| 0 x 6 | 5,1331% | 181 |
| 6 x 12 | 4,9014% | 184 |
| 12 x 18 | 5,1036% | 182 |
| 18 x 24 | 5,1324% | 184 |

The notional principal of the swap is $100 million.

(a) Determine the fixed rate on the two-year interest swap using FRA rates and the following day-count conventions: "30/360", "actual/365" and "actual/360".

(b) Determine the fixed rate on the two-year interest swap under the assumption that the fixed-rate receiver will pay up-front fee equal to 2% of the notional principal ?

**Solution**

(a)

| | $K_t$ | $p_t=d_t-d_{t-1}$ | $_tf_{t-1}$ | $\frac{p_t}{360}$ | $1+{_tf_{t-1}}\frac{p_t}{360}$ | $k_t$ | $a_t$ | $\sum_{t=1}^{T} {_tf_{t-1}}\frac{p_t}{360}K_t a_t$ |
|---|---|---|---|---|---|---|---|---|
| 0 x 6 | 100 | 181 | 5,13% | 0,503 | 102,58% | 102,58% | 97,48% | 2,5 |
| 6 x 12 | 100 | 184 | 4,90% | 0,511 | 102,51% | 105,15% | 95,10% | 4,9 |
| 12 x 18 | 100 | 182 | 5,10% | 0,506 | 102,58% | 107,86% | 92,71% | 7,3 |
| 18 x 24 | 100 | 184 | 5,13% | 0,511 | 102,62% | 110,69% | 90,34% | 9,7 |

The fixed swap rates for semiannual settlements are not equal to fixed swap rates for quarterly settlements.

| | $\sum_{t=1}^{T}\frac{1}{2}K_t a_t$ | $s_{30/360}$ | $\sum_{t=1}^{T}\frac{p_t}{365}K_t a_t$ | $s_{a/365}$ | $\sum_{t=1}^{T}\frac{p_t}{360}K_t a_t$ | $s_{a/360}$ |
|---|---|---|---|---|---|---|
| 0 x 6 | 48,7 | 5,1616% | 48,3 | 5,2044% | 49,0 | 5,1331% |
| 6 x 12 | 96,3 | 5,0869% | 96,3 | 5,0874% | 97,6 | 5,0177% |
| 12 x 18 | 142,6 | 5,1108% | 142,5 | 5,1157% | 144,5 | 5,0456% |
| 18 x 24 | 187,8 | 5,1434% | 188,1 | 5,1370% | 190,7 | 5,0666% |

The fixed swap rate on a two-year interest swap is 5,1434% (30/360 basis), 5,1370% (a/365 basis), and 5,0666% (a/360 basis).

(b)

As the fixed-rate receiver pays an up-front fee, the swap fixed rates are lower.

| | $\sum_{t=1}^{T}\frac{1}{2}K_t a_t$ | $s_{30/360}$ | $\sum_{t=1}^{T}\frac{p_t}{365}K_t a_t$ | $s_{a/365}$ | $\sum_{t=1}^{T}\frac{p_t}{360}K_t a_t$ | $s_{a/360}$ |
|---|---|---|---|---|---|---|
| 0 x 6 | 48,7 | 1,0584% | 48,3 | 1,0672% | 49,0 | 1,0525% |
| 6 x 12 | 96,3 | 3,0099% | 96,3 | 3,0102% | 97,6 | 2,9690% |
| 12 x 18 | 142,6 | 3,7087% | 142,5 | 3,7123% | 144,5 | 3,6614% |
| 18 x 24 | 187,8 | 4,0785% | 188,1 | 4,0734% | 190,7 | 4,0176% |

**Problem 5. Pricing and Valuation of FRA**

The current term structure of WIBOR is

| | |
|---|---|
| 92 -day WIBOR | 5,00% |
| 181 -day WIBOR | 5,20% |

The notional principal is $100 000.

(a) Calculate FRA.

(b) It is 61 days later and the relevant term structure is

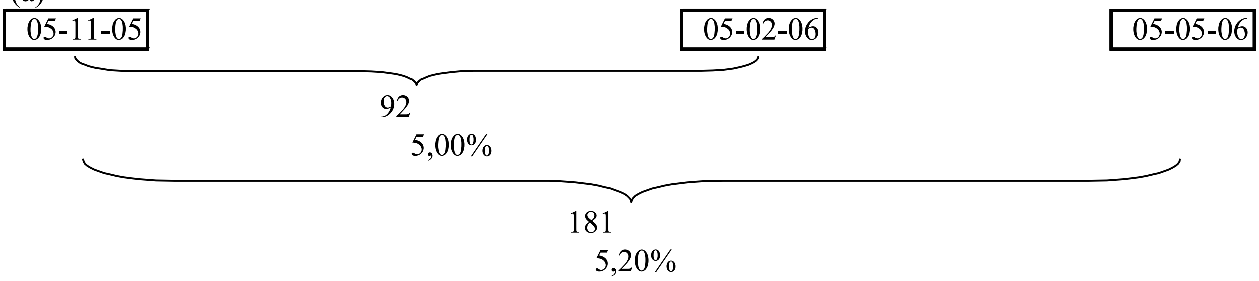
| | |
|---|---|
| 31 -day WIBOR | 4,99% |
| 120 -day WIBOR | 4,84% |

Determine the market value of the FRA.

(c) On the expiration day, 89-day WIBOR is 4,00%. Determine the payment.

**Solution**

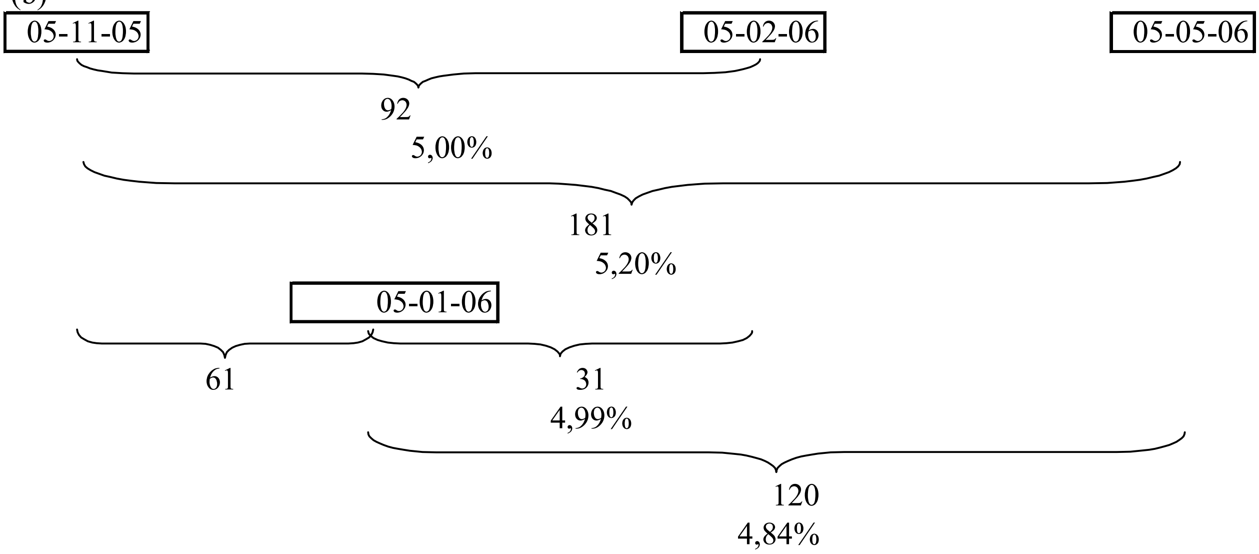
(a)




The FRA rate is:

$$ {}_2f_1 = \left[ \frac{\left(1 + \frac{z_2 t_2}{365}\right)}{\left(1 + \frac{z_1 t_1}{365}\right)} - 1 \right] \frac{365}{t_2 - t_1} \qquad = \qquad 5{,}34\% $$

(b)



The value of FRA will be

$$V_t = \left( \frac{1}{1 + \frac{z_1' t_1'}{365}} - \frac{1 + {}_2f_1 \frac{t_2 - t_1}{365}}{1 + \frac{z_2' t_2'}{365}} \right) K \quad = \quad -137,722$$

or

$${}_2f_t \quad = \quad 4,77\%$$

$$V_t = \left( \frac{{}_2f_t - {}_2f_1}{1 + \frac{z_2' t_2'}{365}} \frac{t_2 - t_1}{365} \right) K \quad = \quad -137,722$$

(c)
At expiration, the payoff is

$$V_t = \left( 1 - \frac{1 + {}_2f_1 \frac{t_2 - t_1}{365}}{1 + \frac{z_2'' t_2'}{365}} \right) K \quad = \quad -323,451$$

or

$$V_t = \left( \frac{z_2'' - {}_2f_1}{1 + \frac{z_2'' t_2'}{365}} \frac{t_2 - t_1}{365} \right) K \quad = \quad -323,451$$

**Problem 6. Pricing and Valuation of IRS**

Consider a one-year interest swap with semiannual payments.
(a) Determine the fixed rate on the swap.
The current structure of WIBOR spot rates is given as follows.

| | | |
|---|---|---|
| Days | 184 | 365 |
| WIBOR | 4,50% | 4,60% |

(b) 153 days later, the term structure is as follows:

| | | |
|---|---|---|
| Days | 31 | 212 |
| WIBOR | 4,99% | 4,77% |

Determine the market value of the swap from the perspective of the party paying the fixed rate and receiving the floatnig rate. Assume the notional principal of $100 000 million.

**Solution**

(a)

$$s_T = \frac{1 - \dfrac{1}{1 + z_T \dfrac{d_T}{365}}}{\sum_{t=1}^{T} \dfrac{\dfrac{d_t - d_{t-1}}{365}}{\dfrac{1}{1 + z_t \dfrac{d_t}{365}}}} = 4{,}55\%$$

| | | | |
|---|---|---|---|
| $d_t$ | 184 | 365 | |
| $z_t$ | 4,50% | 4,60% | |
| $1/(1+z_t d_t/365)$ | 0,9778 | 0,9560 | |
| $d_t$ - $d_{t-1}$ | 184 | 181 | |
| $(d_t - d_{t-1})/365$ | 0,5041 | 0,4959 | Σ |
| $(d_t - d_{t-1})/365/(1+z_t dt/365)$ | 0,4929 | 0,4741 | 0,9670 |

(b)

$$MTM_t = \frac{{}_t f_{t\text{-}1} \dfrac{p_t}{365} + 1}{1 + z_t \dfrac{d_t}{365}} - \sum_{t=1}^{T} \frac{s_T \dfrac{p_t}{365}}{1 + z_t \dfrac{d_t}{365}} - \frac{1}{1 + z_T \dfrac{d_T}{365}}$$

The present value of floating payments

| | |
|---|---|
| Number of days | 184 |
| Floating rate (least reference date) | 4,50% |
| Cash flows | 1,0227 |
| Number of days | 31 |
| Spot rate | 4,99% |
| Present value factor | 0,996 |
| Discounted cash flow | 1,0184 |

The present value of remaining fixed payments:

| | | | |
|---|---|---|---|
| Number of days | 184 | 181 | |
| Swap fixed rate | 4,55% | 4,55% | |
| Cash flows | 0,0229 | 1,0226 | |
| Number of days | 31 | 212 | |
| Spot rate | 4,99% | 4,77% | |
| Present value factor | 0,9958 | 0,9730 | Σ |
| Discounted cash flow | 0,0228 | 0,9950 | 1,0178 |

Difference 0,0006 x 100 000 = 58

# 1.3 Hedging

## 1.3.1 Swap Applications

1. Interest rate gap management
2. Currency gap management
3. Credit risk arbitrage
4. Structured finance
5. Asymmetric information

## 1.3.2 Interest rate management

Duration is a measure of the interest rate sensitivity of an asset value. The durations of the assets and liabilities are obtained as weighted averages of the individual items. A duration gap of zero implies a hedged position against interest rate risk. A conservative bank will attempt to set its duration gap to zero. The positive duration gap means that the bank is exposed to rising interest rates. Implementing the desired duration gap without derivatives is difficult. The long position in a swap or FRA transaction will lengthen liability duration and decrease duration gap. The negative duration gap means that the bank is exposed to falling interest rates. The short position in a swap or FRA transaction will lengthen asset duration and increase duration gap.

The notional capital of a FRA or swap transaction is derived using the following equation:

(5) $L_B A + D_S K = L_P (A+K)$

where:
$L_B$ – the existing duration gap,
A – assets
$D_S$ – swap's duration,
K – notional principal,
$L_P$ – required duration gap.

Solving for K we obtain:

(6) $$K = \frac{A(L_B - L_P)}{L_P - D_S}$$

The duration of the FRA or swap transaction should be higher than the required duration gap.

A swap used to restructure assets is referred to asset swap. A swap used to restructure liabilities is called a liability swap. Both of them may imply increase or decrease of a duration gap (figure 1 ).

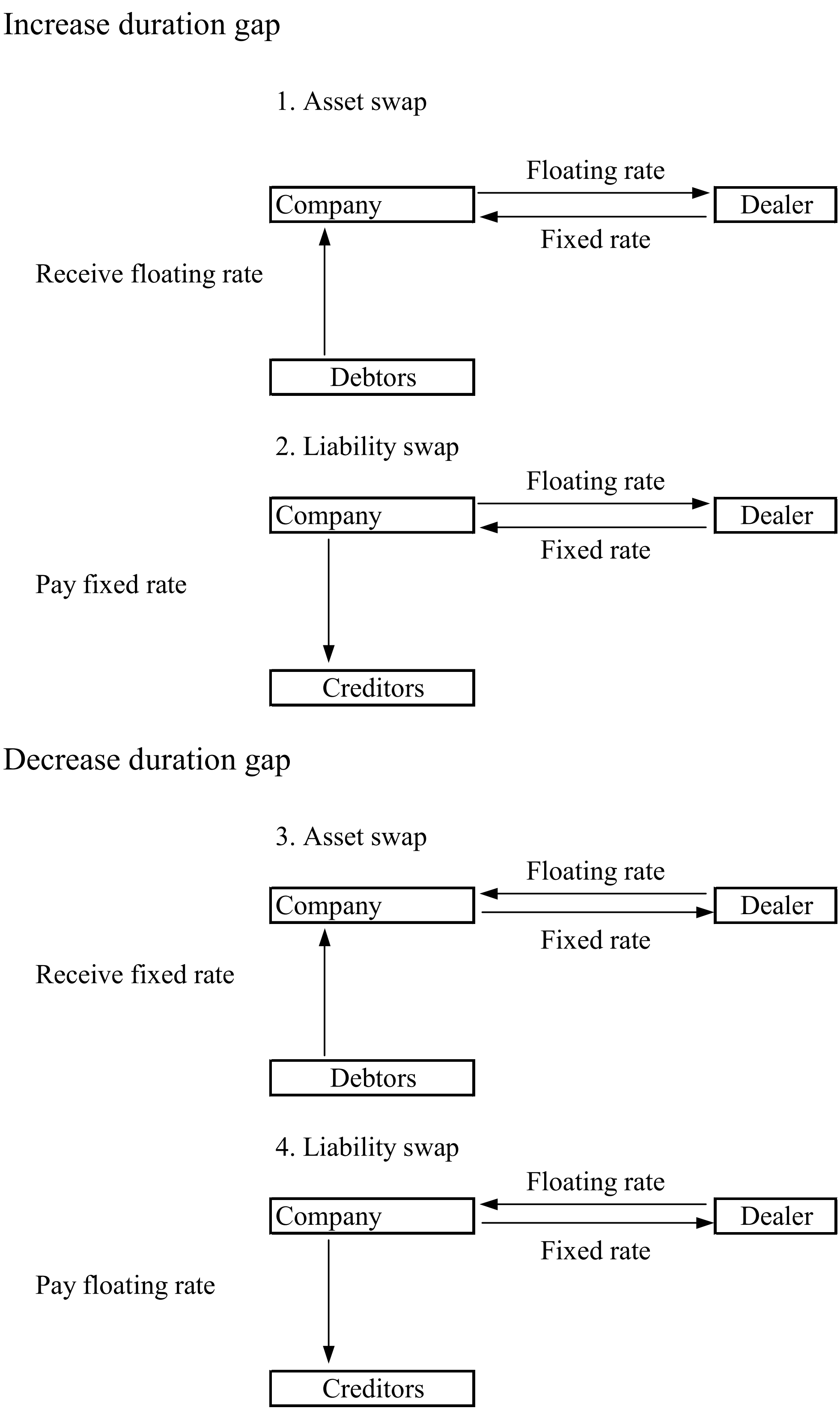



Figure 1. Asset Swap and Liability Swap

**Problem 7. Duration Gap**

Consider the following balance sheet

| Assets | $ million | Duration | Claims | $ million | Duration |
|---|---|---|---|---|---|
| Group I | 100 | 0,00 | Group I | 600 | 1,00 |
| Group II | 200 | 2,00 | Group II | 100 | 2,00 |
| Group III | 100 | 3,00 | Group III | 100 | 3,00 |
| Group IV | 100 | 5,00 | Group IV | 100 | 4,00 |
| Group V | 500 | 6,00 | Equity | 100 | 0,00 |
| Total | 1000 | 4,20 | Total | 1000 | 1,50 |

(a) Calculate the duration gap. What is the interest rate exposure ?
What is the impact of a 300-basis-point increase on the equity ?
(b) Calculate the notional principal of a swap transaction to set the duration gap to zero.
Assume that the duration for a swap transaction is -4,5 years.
Show a new balance sheet.
(c) Calculate the notional principal of a swap transaction to set the required duration gap equal to 1,0 year. Show a new balance sheet.

**Solution**

(a)

Duration gap is equal to 4,20 - 1,50 = 2,70.

The positive duration gap means that the market value of equity would decline if interest rates rose.

The rise in interest rates of 3 basis points will decrease the market value of equity approximately by 2,70 * 3% * 1000 = 81.

(b)

The notional principal should be 1000 * (2,70 - 0,00) : (0,00 - (-4,50)) = 600.

| | Assets | | Claims on assets | |
|---|---|---|---|---|
| | $ million | Duration | $ million | Duration |
| balance sheet | 1000 | 4,20 | 1000 | 1,50 |
| swap | 600 | 1,00 | 600 | 5,50 |
| total | 1600 | 3,00 | 1600 | 3,00 |

(c)

The notional principal should be 1000 * (2,70 - 1,00) : (1,00 - (-4,50)) = 309.

| | Assets | | Claims on assets | |
|---|---|---|---|---|
| | $ million | Duration | $ million | Duration |
| balance sheet | 1000 | 4,20 | 1000 | 1,50 |
| swap | 309 | 1,00 | 309 | 5,50 |
| total | 1309 | 3,44 | 1309 | 2,44 |