1. Interest Rate Exposure. Stochastic Methods. Interest Rate Gap. Duration

1.1 Exposure

Book value
Book value + commitments
Market Value
Economic value (valuation models)

Valuation Models

1. Discrete models
   a. Binomial
   b. One-path (DCF models)
      i. Traditional – one discount rate
      ii. Non arbitrage – many discount rates (spot rates)
2. Continuous models (BSM) – what can be valued ?

Option pricing (binomial and BS) models are used to value assets with embedded options. Binomial model is used to value callable bonds, putable bonds, floating rate notes, and structured notes in which the cash flows are based on interest rate. Simulation methods are used to value assets in assumed risky environment. The Monte Carlo simulation is used to value mortgage-backed securities and certain type of asset-backed securities in which the cash flows are based on interest rate path.

1. Traditional Valuation:

$$PV = \frac{CF_1}{(1+RRR)^1} + \frac{CF_2}{(1+RRR)^2} + \ldots + \frac{CF_n}{(1+RRR)^n} + \frac{CV_n}{(1+RRR)^n}$$

2. Non Arbitrage Valuation

$$PV = \frac{CF_1}{(1+RRR_1)^1} + \frac{CF_2}{(1+RRR_2)^2} + \ldots + \frac{CF_n}{(1+RRR_n)^n} + \frac{CV_n}{(1+RRR_n)^n}$$

Differences between two approaches are shown in the following table (bond as an example):

	Fixed rate	Floating rate
Traditional approach	$P = \frac{cB}{(1+YTM)^1} + \frac{cB}{(1+YTM)^2} + \ldots + \frac{cB+B}{(1+YTM)^T}$	$P = \frac{z_1 B}{(1+YTM)^1} + \frac{{}_2f_1 B}{(1+YTM)^2} + \ldots + \frac{{}_Tf_{T-1} B + B}{(1+YTM)^T}$
Arbitrage - free approach	$P = \frac{cB}{(1+z_1)^1} + \frac{cB}{(1+z_2)^2} + \ldots + \frac{cB+B}{(1+z_T)^T}$	$P = \frac{z_1 B}{(1+z_1)^1} + \frac{{}_2f_1 B}{(1+z_2)^2} + \ldots + \frac{{}_Tf_{T-1} B + B}{(1+z_T)^T}$

P - price, c - coupon rate, B - face value, YTM - yield to maturity, z - spot rate, f - forward rate.

The traditional valuation methodology discounts every cash flow of an asset by the same discount rate. The arbitrage-free approach values an asset with each cash flow discounted at its unique discount rate (spot rate).

Problem 1. Bond price, YTM and maturity

Consider a coupon bond with a face value $1000 paying an annual coupon of 10%.
Required:
(a) Calculate the market price of a bond as a result of changes in market yield in the range 8-12% and for different maturities: 1 year, 5 years, 10 years, 15 years 20 years, 25 years and 30 years.
(b) Show sensitivity of return changes on above yields and maturities.

Solution

Ad 1.

Market prices of a bond as a result of changes in market yield for the assumed maturities are:

	Maturity (years)						
Rate	1	5	10	15	20	25	30
8%	1018,5	1079,9	1134,2	1171,2	1196,4	1213,5	1225,2
9%	1009,2	1038,9	1064,2	1080,6	1091,3	1098,2	1102,7
10%	1000,0	1000,0	1000,0	1000,0	1000,0	1000,0	1000,0
11%	991,0	963,0	941,1	928,1	920,4	915,8	913,1
12%	982,1	927,9	887,0	863,8	850,6	843,1	838,9

Ad 2.

The percentage price changes are following:

	Maturity (years)						
Rate	1	5	10	15	20	25	30
8%	1,9%	8,0%	13,4%	17,1%	19,6%	21,3%	22,5%
9%	0,9%	3,9%	6,4%	8,1%	9,1%	9,8%	10,3%
10%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
11%	-0,9%	-3,7%	-5,9%	-7,2%	-8,0%	-8,4%	-8,7%
12%	-1,8%	-7,2%	-11,3%	-13,6%	-14,9%	-15,7%	-16,1%

1.2 Stochastic models

1.2.1 General equilibrium Term Structure Models

Cox, Ingersoll and Ross [1985] derived stochastic process of interest rates as[1]:

(1) $$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dz$$

where

r – the current spot rate,

θ - the central location or long term value,

κ - the pull parameter that governs the speed at which the spot rate is drawn back to the long term value,

σ - volatility,

dt – a small change in time,

dz - standard, one-dimensonal Wiener process.

[1] J.C.Cox, J.E. Ingersoll, Jr., Stephen Ross, *An Intertemporal General Equilibrium Model of Asset Prices*, „Econometrica”, 1985, vol. 53, no. 2 s. 363-384 oraz tychże *Theory of the Term Structure of Interest Rates*, „Econometrica”, 1985, vol. 53, no. 2 s. 385-407.

1.2.2 Arbitrage-free Modelling

These models take a linear stochastic differential equation of the general form

(2) $dr = \mu(r,t)dt + \sigma(r,t)dz$

gdzie:
r - the current spot rate,
μ - the drift term,
σ - volatility,
dt - a small change in time,
dz - standard, one-dimensonal Wiener process.

Ho and **Lee** provided one of the first arbitrage-free models of the term structure. The stochastic differential equation was[2]:

(3) $dr = \mu(t)dt + \sigma dz$

They assumed that interest rate shocks were normally distributed. The mean μ was selected to match exactly the current structure. The volatility parameter σ was fixed. The disadvantage of this model is that negative interest rates are possible. In binomial lattice, the up and down jumps were expressed as

(4) $r_u = r_0 + \mu(ts) + \sigma\sqrt{ts}$

(5) $r_d = r_0 + \mu(ts) - \sigma\sqrt{ts}$

gdzie:
ts – time step.

In the lognormal model (the same assumptions as in the BSM)) stochastic differential equation was:

(6) $dr = \mu(t)\, r\, dt + \sigma\, r\, dz$

or (using Ito's lemma):

(7) $d\ln(r) = \left[\mu(t) - \frac{\sigma^2}{2}\right]dt + \sigma dz$

This model excludes possibility of obtaining negative, but still ignores the strong mean-reverting process. The interest rate volatility is proportional to rate level but is still independent in time. The up and down jumps were expressed as follows

(8) $r_u = r_0 \exp^{\mu(ts)+\sigma\sqrt{ts}}$

(9) $r_d = r_0 \exp^{\mu(ts)-\sigma\sqrt{ts}}$

Black, Derman and Toy adopted a lognormal distribution and introduced time-varying rate volatility. The stochastic differential equation was[3]:

(10) $dr = \mu(t)\, r\, dt + \sigma(t)\, r\, dz$

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[2] T.S.Y. Ho, S. Lee, *Term Structure Movements and Pricing Interest Rate Contingent Claims*, „Journal of Finance", 1986, vol. 41, no. 5.

[3] F. Black, E. Derman, W. Toy, *A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options*, „Financial Analysts Journal", January 1990, vol. 46, no. 1, s. 33-39.

The up and down jumps were expressed as follows:

(11) $$r_u = r_0 \exp^{\mu(ts)+\sigma(ts)\sqrt{ts}}$$

(12) $$r_d = r_0 \exp^{\mu(ts)-\sigma(ts)\sqrt{ts}}$$

Blacka and **Karasiński** extended the previous model by explicitly incorporating a mean reversion parameter κ[4]

(13) $$dr = \kappa(t)\,\{\ln[\mu(t)] - \ln[r(t)]\}\,r\,dt + \sigma(t)\,r\,dz$$

where:

κ - mean reversion parameter.

The up and down jumps were expressed as follows:

(14) $$r_u = r_0 \exp^{\kappa(ts)[\mu(ts) - r(ts)]ts+\sigma(ts)\sqrt{ts}}$$

(15) $$r_d = r_0 \exp^{\kappa(ts)[\mu(ts) - r(ts)]ts-\sigma(ts)\sqrt{ts}}$$

Hull and **White'a**[5] introduced general framework:

(16) $$dx = a\left[\frac{\theta(t)}{a} - x\right]dt + \sigma\,dz$$

Heatha, Jarrow and **Morton**[6] assumes that each forward rate may change based on its own sensitivities to the underlying factors. The term structure may change and twist in a wide variety of ways. The stochastic differential equation of the family of forward rates can be expressed as

(17) $$df(T) = \int_0^t \mu(v,T,\omega)dv + \sum_{i=1}^{n}\int_{i=1}^{t}\sigma_i(v,T,\omega)dW_i(v)$$

To compare stochastic differential models the following general formula may be used:

(18) $$dr(t) = [\alpha_1(t) + \alpha_2(t)r(t) + \alpha_3(t)\ln(r(t))]d(t) + [\beta_1(t) + \beta_2(t)r(t)]^{\gamma}\,dz$$

Tabela 1. Interest Rate Term Structure Models

Autor	α_1	α_2	α_3	β_1	β_2	γ
Merton (1974)	☺			☺		1
Vasicek (1977)	☺	☺		☺		1
Brennan-Schwartz (1979)	☺	☺			☺	1
Cox-Ingersoll-Ross (1980)					☺	1,5
Cox-Ingersoll-Ross (1985)	☺	☺			☺	0,5
Ho-Lee (1986)	☺			☺		1
Salomon Brothers		☺			☺	1
Black-Derman-Toy		☺			☺	1
Black-Karasiński (1991)		☺	☺		☺	1
Pearson-Sun (1994)	☺	☺		☺	☺	0,5

Source: Table based on idea presented in the book: A.Weron, R.Weron, *Inżynieria finansowa. Wycena instrumentów pochodnych. Symulacje komputerowe. Statystyka rynku*, Wydawnictwa Naukowo-Techniczne, Warszawa 1998, s. 211.

[4] Por. F. Black i P. Karasiński, *Bond and Option Pricing When Short Rates Are Lognormal*, „Financial Analysts Journal", 1991, vol. 47, no. 4.

[5] J. Hull i A. White, *Using Hull-White Interest Rate Trees*, „Journal of Derivatives", 1996, vol. 3, no. 3.

[6] D.Heath, R.Jarrow, A.Morton, *Bond Pricing and the Term Structure of Interest Rates: A New Methodology*, „Econometrica", 1996, vol. 60, no. 1, s. 77-105.

1.3 Interest Rate Gap. Duration

Maturity Gap

Problem 2. Maturity Gap

A bank invests $100 million in 3-year, 10% fixed rate bonds (assume these are all assets)
In the same time, it issuses $90 million in 1-year, 10% percent fixed rate bonds (assume these are all liabilities).
(a) Show the market value of equity assuming interest rates change in the range 8-17 percent.
(b) Is it the interest rate risk eliminated when maturity of the issued bonds is 3 years ?

Solution

(a)

	Assets	Liabilities
Maturity	3	1
Coupon rate	10%	10%
Face value	100	90

YTM	Assets	Liabilities	Equity	Δ Equity
8%	105,15	91,67	13,49	3,49
9%	102,53	90,83	11,71	1,71
10%	100,00	90,00	10,00	0,00
11%	97,56	89,19	8,37	-1,63
12%	95,20	88,39	6,80	-3,20
13%	92,92	87,61	5,31	-4,69
14%	90,71	86,84	3,87	-6,13
15%	88,58	86,09	2,50	-7,50
16%	86,52	85,34	1,18	-8,82
17%	84,53	84,62	-0,08	-10,08

(b)

	Assets	Liabilities
Maturity	3	3
Maturity	10%	10%
Face value	100	90

YTM	Assets	Liabilities	Equity	Δ Equity
8%	105,15	94,64	10,52	0,52
9%	102,53	92,28	10,25	0,25
10%	100,00	90,00	10,00	0,00
11%	97,56	87,80	9,76	-0,24
12%	95,20	85,68	9,52	-0,48
13%	92,92	83,62	9,29	-0,71
14%	90,71	81,64	9,07	-0,93
15%	88,58	79,73	8,86	-1,14
16%	86,52	77,87	8,65	-1,35
17%	84,53	76,08	8,45	-1,55

Maturity gap is the difference between the weighted average maturities of assets and liabilities. It may be greater than, equal to or less than zero. A rise in interest rates reduces the market values of assets and liabilities. If the maturity of assets is longer than maturity of liabilities (maturity gap is greater than zero), the market value of assets falls by more than the market value of liabilities. The equity of the bank declines.

Maturity matching does not perfectly immunize or protect against interest rate risk. Maturity gap is an incomplete measure of interest rate risk. Duration gap is much more accurate risk measure.

Duration Gap

Duration is the average life of an asset, or more exactly, the weighted average time to maturity using the relative present values of the cash flows as weights. Duration is measured in years.

The modified duration is a measure of the interest sensitivity of an asset's price. The larger is the modified duration, the price of the asset is more sensitive.

You can calculate the modified duration (instrument with annual cash flows) using the general formula:

$$(19) \qquad D = \frac{1}{(1+i)} \sum_{t=1}^{T} \frac{t \dfrac{CF_t}{(1+i)^t}}{W}$$

<u>EXCEL FUNCTIONS</u>

DURATION(settlement, maturity, coupon, yld, frequency, basis)

Returns the annual duration of a security with periodic interest payments.
Duration is defined as the weighted average of the present value of the cash flows,
and is used as a measure of a bond price's response to changes in yield.

MDURATION(settlement, maturity, coupon, yld, frequency, basis)

Returns the modified Macauley duration for a security with an assumed par value of $100.
MDURATION = DURATION / (1 + market yield/coupon payments per year)

Settlement is the security's settlement date, expressed as a serial date number.
Maturity is the security's maturity date, expressed as a serial date number.
Coupon is the security's annual coupon rate.
Yld is the security's annual yield.
Frequency is the number of coupon payments per year. For annual payments, frequency = 1;
for semiannual, frequency = 2; for quarterly, frequency = 4.
Basis is the type of day count basis to use.

Basis	Day count basis
0 or omitted	US (NASD) 30/360
1	Actual/actual
2	Actual/360
3	Actual/365
4	European 30/360

Problem 3. Duration

Consider a Eurobond with an annual coupon that pays $1000 in 5 years.

Required

(a) Calculate the duration and the modified of the bonds with following characteristics:

I. Coupon 10% and YTM 10%.

II. Coupon 10% and YTM 15%.

III. Coupon 10% and YTM 5%.

(b) Show the sensitivity analysis of the modified duration on changes in coupon rate and YTM in the range 2-20%.

Solution

(a)

Coupon rate		10%	YTM		15%

Year	Cash Flow	Disc. factor	DCF	Weight	Year * Weight
1	100	0,9091	90,91	0,0909	0,0909
2	100	0,8264	82,64	0,0826	0,1653
3	100	0,7513	75,13	0,0751	0,2254
4	100	0,6830	68,30	0,0683	0,2732
5	1100	0,6209	683,01	0,6830	3,4151
			1000,00	1,0000	4,1699

Duration is 4,17 years.

Modified duration is 4,17 * 1/(1+10%) = 3,79.

Coupon rate		10%	YTM		15%

Year	Cash Flow	Disc. factor	DCF	Weight	Year * Weight
1	100	0,8696	86,96	0,1045	0,1045
2	100	0,7561	75,61	0,0908	0,1817
3	100	0,6575	65,75	0,0790	0,2370
4	100	0,5718	57,18	0,0687	0,2748
5	1100	0,4972	546,89	0,6570	3,2851
			832,39	1,0000	4,0829

Duration is 4,08 years.

Modified duration is 4,08 * 1/(1+15%) = 3,55.

Coupon rate		10%	YTM		5%

Year	Cash Flow	Disc. factor	DCF	Weight	Year * Weight
1	100	0,9524	95,24	0,0783	0,0783
2	100	0,9070	90,70	0,0746	0,1491
3	100	0,8638	86,38	0,0710	0,2130
4	100	0,8227	82,27	0,0676	0,2705
5	1100	0,7835	861,88	0,7085	3,5425
			1216,47	1,0000	4,2535

(b)

Sensitivity analysis

Coupon	D
2%	4,3288
10%	3,7908
20%	3,4510

YTM	D
2%	4,2175
10%	3,7908
20%	3,3278

Problem 4. Modified Duration. Sensitivity Analysis

Modified Duration. Sensitivity Analysis

Maturity 1 year

Yield to maturity (rows) / Coupon rate (columns)

Annual cash flows

	0%	1%	5%	10%	20%	100%
0%	1,00	1,00	1,00	1,00	1,00	1,00
1%	0,99	0,99	0,99	0,99	0,99	0,99
5%	0,95	0,95	0,95	0,95	0,95	0,95
10%	0,91	0,91	0,91	0,91	0,91	0,91
20%	0,83	0,83	0,83	0,83	0,83	0,83
100%	0,50	0,50	0,50	0,50	0,50	0,50

Quarterly cash flows

	0%	1%	5%	10%	20%	100%
0%	1,00	1,00	0,98	0,97	0,94	0,81
1%	1,00	0,99	0,98	0,96	0,93	0,81
5%	0,99	0,98	0,97	0,95	0,92	0,80
10%	0,98	0,97	0,96	0,94	0,91	0,79
20%	0,95	0,95	0,93	0,92	0,89	0,76
100%	0,80	0,79	0,78	0,76	0,72	0,59

	Y	Y	Y	Y	Y
		Y	Y	Y	Y
				Y	Y
					Y
					Y

Maturity 10 years

Annual cash flows

	0%	1%	5%	10%	20%	100%
0%	10,00	9,59	8,50	7,75	7,00	5,91
1%	9,90	9,47	8,34	7,58	6,83	5,76
5%	9,52	9,00	7,72	6,92	6,18	5,20
10%	9,09	8,43	6,96	6,14	5,44	4,58
20%	8,33	7,32	5,54	4,77	4,19	3,58
100%	5,00	1,35	1,07	1,03	1,01	1,00

Quarterly cash flows

	0%	1%	5%	10%	20%	100%
0%	10,00	9,56	8,38	7,56	6,75	5,57
1%	9,98	9,50	8,27	7,44	6,62	5,46
5%	9,88	9,28	7,83	6,94	6,12	5,03
10%	9,76	8,96	7,22	6,28	5,48	4,52
20%	9,52	8,11	5,85	4,94	4,29	3,62
100%	8,00	1,09	1,02	1,01	1,00	1,00

	Y	Y	Y	Y	Y
		Y	Y	Y	Y
				Y	Y
					Y
Y	Y	Y	Y		

Maturity 100 years

Annual cash flows

	0%	1%	5%	10%	20%	100%
0%	100,0	75,25	58,75	55,00	52,86	50,99
1%	99,01	63,03	47,92	45,09	43,55	42,26
5%	95,24	22,07	19,85	19,56	19,42	19,30
10%	90,91	10,05	10,01	10,00	10,00	9,99
20%	83,33	5,00	5,00	5,00	5,00	5,00
100%	50,00	1,00	1,00	1,00	1,00	1,00

Quarterly cash flows

	0%	1%	5%	10%	20%	100%
0%	100,0	75,06	58,44	54,66	52,50	50,62
1%	99,75	63,17	47,88	45,02	43,47	42,17
5%	98,77	22,00	19,86	19,59	19,45	19,34
10%	97,56	10,04	10,00	10,00	10,00	10,00
20%	95,24	5,00	5,00	5,00	5,00	5,00
100%	80,00	1,00	1,00	1,00	1,00	1,00

	Y	Y	Y	Y	Y
		Y	Y	Y	Y
	Y				
	Y	Y			
	Y	Y	Y		
					Y

Do high yield bonds have high or low durations ?
Do high coupon bonds have high or low durations ?
Does duration is more sensitive to yield or coupon rate ?
Do more frequent cash flows always reduce modified duration ?

Important features of the modified duration (True or False)

1. Duration increases with the maturity.
2. Duration increases as yield decreases.
3. Duration increases as the coupon rate decreases.

Table 2. Modified duration and convexity of bonds

Bonds	Modified duration	Convexity
Coupon bonds (annual interests)	$D = \frac{B\left(c[1+i]\left[(1+i)^T - 1\right] + iT[i-c]\right)}{Pi^2(1+i)^{T+1}}$	$C = \frac{2B\left(c[1+i]^2\left[(1+i)^T - 1\right] - ciT[1+i] + i^2T[T+1]\frac{[i-c]}{2}\right)}{Pi^3(1+i)^{T+2}}$
Zero coupon bonds	$D = \frac{T}{(1+i)}$	$C = \frac{T(T+1)}{(1+i)^2}$
Consol bonds	$D = \frac{1}{i}$	$C = \frac{2}{i^2}$

where: P - price, i – annualized yield, c – coupon rate, B – face value, T - maturity.

Duration Model

(20) $$\frac{\Delta W}{W} = -D\,\Delta i$$

Duration and convexity model

(21) $$\frac{\Delta P}{P} = -D\Delta i + \frac{1}{2}C(\Delta i)^2$$

Macaulay Duration Model

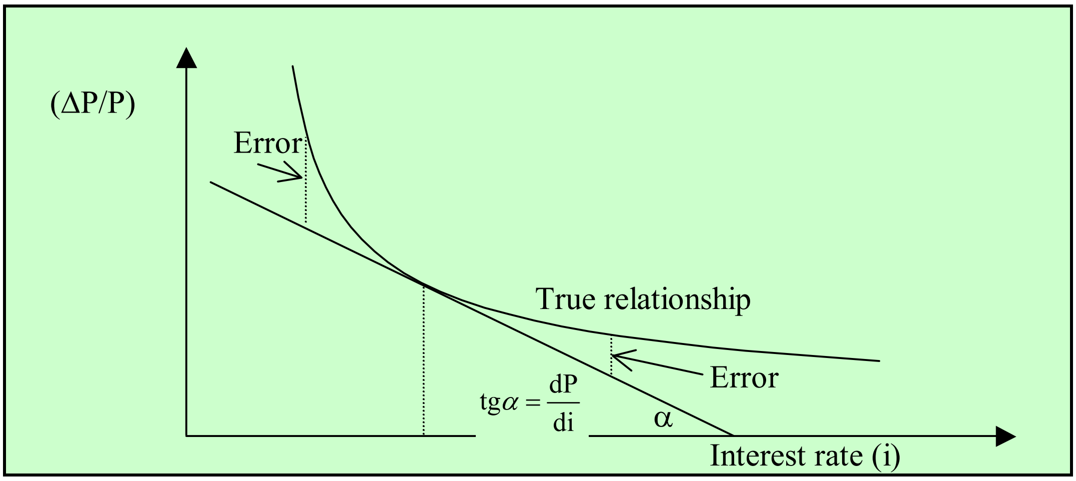
(22) $$\frac{\Delta W}{W} = -D_z\left(\frac{\Delta i}{1+i}\right) = -\frac{D_z}{1+i}\cdot\Delta i$$

(23) $$D = \frac{D_z}{1+i}$$

Effective duration and effective convexity

(24) $$D = \frac{V_- - V_+}{2V_0\Delta y}$$

(25) $$C = \frac{V_- - V_+ - 2V_0}{2V_0(\Delta y)^2}$$



Problem 5. Modified Duration and Convexity

A bond is currently selling for $950 and has 15 years left to maturity and a par value of $1000. The bond has a 10% coupon (payable annually).
(a) Calculate the YTM, the duration, the modified duration and the convexity.
(b) Calculate the impact of a +-1%, +-5% changes in interest rates on the price of the bond. Use duration, duration and convexity approximation and the exact price using bond valuation.

Solution

(a)

Year	Cash Flow	DCF	Weight	Duration	Convexity
t	CF_t	$\frac{CF_t}{(1+i)^t}$	$\frac{CF_t}{P(1+i)^t}$	$\frac{tCF_t}{P(1+i)^t}$	$\frac{t(t+1)CF_t}{P(1+i)^{t+2}}$
0	-950				
1	100	90,35	0,0951	0,0951	0,1553
2	100	81,63	0,0859	0,1718	0,4208
3	100	73,75	0,0776	0,2329	0,7604
4	100	66,63	0,0701	0,2805	1,1450
5	100	60,20	0,0634	0,3168	1,5518
6	100	54,39	0,0573	0,3435	1,9628
7	100	49,14	0,0517	0,3621	2,3644
8	100	44,40	0,0467	0,3739	2,7466
9	100	40,11	0,0422	0,3800	3,1018
10	100	36,24	0,0381	0,3815	3,4252
11	100	32,74	0,0345	0,3791	3,7135
12	100	29,58	0,0311	0,3737	3,9651
13	100	26,73	0,0281	0,3657	4,1795
14	100	24,15	0,0254	0,3558	4,3570
15	1100	239,98	0,2526	3,7891	49,4870
	YTM	P		Duration	Convexity
	10,68%	950,00	1,0000	8,2016	83,3362

Internal rate of return (YTM) is 10,68%. Duration is 8,20 years.
Modified duration is 8,20 * 1/(1+10,7%) = 7,41. Convexity is 83,34.

Ad 2.

			Relative change in price $\Delta P/P$		
Δi	i	Market Price	$\frac{\Delta P}{P} = -D\Delta i$	$-D\Delta i + \frac{1}{2}C(\Delta i)^2$	Price Formula
-5%	5,68%	1428,07	37,05%	47,47%	50,32%
-1%	9,68%	1024,54	7,41%	7,83%	7,85%
0%	10,68%	950,00	0,00%	0,00%	0,00%
1%	11,68%	883,39	-7,41%	-6,99%	-7,01%
5%	15,68%	678,37	-37,05%	-26,63%	-28,59%

Matching the duration of an asset to the investor's target horizon immunizes it against interest rate risk.

Duration can be calculated for assets or liabilities as a market value weighted average of the individual durations of all items. The duration gap is just a difference between the duration of asset portfolio and the duration of liability portfolio.

(26) $D_A = w_{A1}D_{A1} + w_{A2}D_{A2} + ... + w_{An}D_{An}$

(27) $D_L = w_{L1}D_{L1} + w_{L2}D_{L2} + ... + w_{Ln}D_{Ln}$

(28) $\Delta A = - D_A \cdot A \cdot \Delta y$

(29) $\Delta L = - D_L \cdot L \cdot \Delta y$

(30) $\Delta E = \Delta A - \Delta L = [- D_A \cdot A + D_L \cdot L]\Delta y$

(31) $\Delta E = -[D_A - D_L \cdot k] \cdot A \cdot \Delta y$

$k = \frac{L}{A}$ - is a leverage measure.

(32) $\Delta E = -[D_A - D_P] \cdot A \cdot \Delta y$

Problem 6. Duration Gap

A bank invests $100 million in 3-year, 10% percent fixed rate bonds (assume these are all assets). In the same time, it issuses $90 million in 10% percent fixed rate bonds (these are all liabilities).
(a) Calculate the appropriate duration of its liabilities to match the duration of assets.
What should be the maturity of the issued bonds ?
(b) Show the market value of equity assuming interest rates change in the range 8-17 percent.
Is it the interest rate risk completely eliminated ?

Solution

(a)

The duration of the bank's assets is 2,5. Duration gap should be equal to 0.

$$D_A - kD_L = 0$$

Solving for the duration of liabilities, D_L = 2,78.
The maturity of the issued bonds may be calculated using iterations.
If the duration gap is zero, the maturity of the issued bonds will be 3,28.

(b)

	Assets	Liabilities
Maturity	3	3,28
Coupon	10%	10%
Face Value	100	90

YTM	Assets	Liabilities	Equity	Δ Equity	D_A	D_L	k	Gap
8%	105,15	95,02	10,14	0,14	2,52	2,79	0,9	0,002
9%	102,53	92,46	10,07	0,07	2,51	2,78	0,9	0,001
10%	100,00	90,00	10,00	0,00	2,50	2,78	0,9	0,000
11%	97,56	87,63	9,93	-0,07	2,49	2,77	0,9	-0,001
12%	95,20	85,34	9,85	-0,15	2,48	2,76	0,9	-0,002
13%	92,92	83,14	9,78	-0,22	2,47	2,75	0,9	-0,003
14%	90,71	81,02	9,70	-0,30	2,46	2,74	0,9	-0,004
15%	88,58	78,97	9,61	-0,39	2,45	2,73	0,9	-0,005
16%	86,52	76,99	9,53	-0,47	2,44	2,72	0,9	-0,006
17%	84,53	75,09	9,45	-0,55	2,44	2,71	0,9	-0,006

Duration matching does not fully protect against interest rate risk.

Repricing Gap

Repricing gap is the difference between those assets whose interest rates will be repriced (interest rate sensitive assets) and liabilities whose interest rates will be repriced (interest rate sensitive liabilities) over some future period. It is used to calculate net interest income change due to interest rate changes at different maturity buckets.

Problem 7. Repricing Gap

Use the following Interest Rate Risk Reporting Schedule to answer questions a and b.

	Sensitive (cumulative)				Nonsensitive	Total
	within 1M	within 3M	within 6M	within 1R		
Assets						
Cash					15000	15000
Short-term instruments	5000	5000	5000	5000		5000
Investment securities	2000	3000	4000	6000	24000	30000
Loans	32000	35000	50000	60000	40000	100000
Total assets	39000	43000	59000	71000	79000	150000
Liabilities and Capital						
Demand deposits					30000	30000
Short-term deposits	30500	31000	35000	39000	10000	49000
Passbook savings					10000	10000
CDs	3000	10000	20000	30000	2000	32000
Public and other deposits	500	2000	3000	10000	5000	15000
Short-term borrowing	3500	4000	4000	4000		4000
Shareholder equity					10000	10000
Total liabilities and capital	37500	47000	62000	83000	67000	150000

(a) Calculate repricing gap and interst-sensitivity ratio for all periods.
(b) Calculate the impact on net interest income if interest rates change +-1% for the four repricing gaps.

Solution

(a)

	within 1M	within 3M	within 6M	within 1R
Interest-sensitivity gap	1500	-4000	-3000	-12000
Interest-sensitivity ratio	1,04	0,91	0,95	0,86

(b)

Change in net interest income, when R+1%	15	-40	-30	-120
Change in net interest income, when R-1%	-15	40	30	120