3. Interest Rate Risk. Term Structure of Interest Rates. Conversions

Investors like to think in terms of annualized interest rates, but for most money market instruments it is not the case.

Figure 1. Interest rates conversions

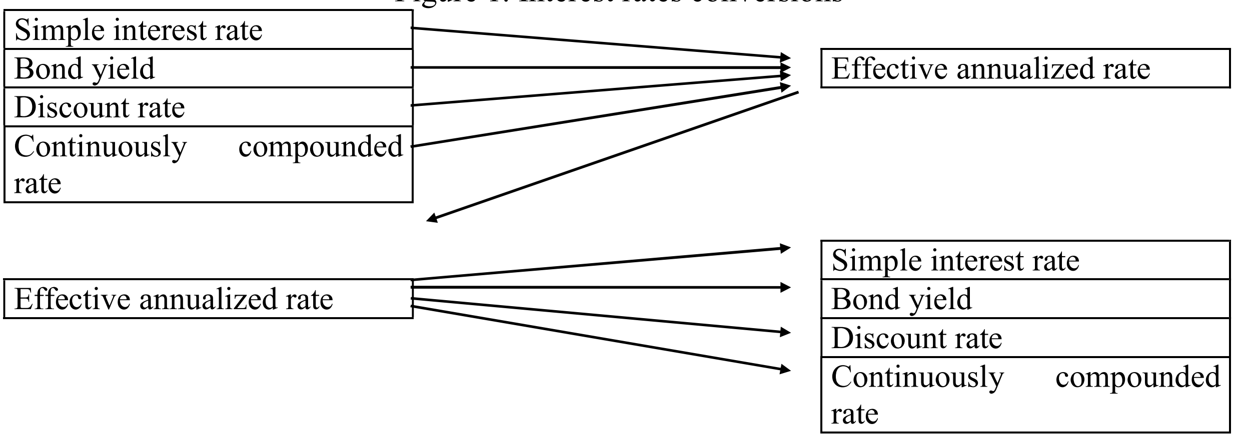


Table 1. Effective rates

	Effective Annual Rate	Equivalent Rate
Simple interest rate	$i=\left(1+\frac{rt}{365}\right)^{\frac{365}{t}}-1$	$r=\frac{365}{t}\left[(1+i)^{\frac{t}{365}}-1\right]$
Bond yield with semiannual payments	$i=\left(1+\frac{y}{2}\right)^{2}-1$	$y=2\left[(1+i)^{\frac{1}{2}}-1\right]$
Discount rate	$i=\left(\frac{1}{1-\frac{dt}{360}}\right)^{\frac{365}{t}}-1$	$d=\frac{360}{t}\left[1-\frac{1}{(1+i)^{\frac{t}{365}}}\right]$ or $d=\frac{r}{1+\frac{rt}{360}}$
Continuously compounded rate	$i=e^{c}-1$	$c=\ln(1+i)$

Source: ZM.

Money market yield

A simple interest or money market yield (r) is used for most deposits, loans and many other instruments. The simple rate is an annual rate used to calculate interest payments. Interest payments are calculated as

(1) $$\text{Interest Payment} = r\frac{t}{365}\ \text{Principal}$$

But r is not an effective annualized rate. The effective annualized rate is

(2) $$i = \left(1 + \frac{rt}{365}\right)^{\frac{365}{t}} - 1$$

and it varies during a year because of changes in the number of days (t) in a period.

Problem 1. Money market yield

If you deposit $100 in the bank today and it earns interest at a rate of 10% compounded monthly.

(a) How much will be in the account 12 months from today if you calculate interest on a a/365 basis ?"
What is the effective annualized interest rate ?

(b) What is the effective annualized interest rate on a 30/360 basis ?

(c) What is the equivalent continuously compounded rate ?

Solution

(a)

Month	No of days	Balance	Effective rate
1	31	100,849315	10,470434%
2	28	101,622954	10,474925%
3	31	102,486053	10,470434%
4	30	103,328404	10,471930%
5	31	104,205988	10,470434%
6	30	105,062475	10,471930%
7	31	105,954787	10,470434%
8	31	106,854676	10,470434%
9	30	107,732934	10,471930%
10	31	108,647926	10,470434%
11	30	109,540923	10,471930%
12	31	110,471270	10,470434%

The effective rate for the whole year is equal 10,471270%

(b)

On a 30/360 basis, the effective rate is (1+10%/12)^12= 10,471307%

(c)

The equivalent continuously compounded rate is equal to ln(1+10,47%) = 9,96%.

Bond Yield

Annual Payments

The price of a bond is given by

(3) $$P_0 = \sum_{t=1}^{n} \frac{cB}{(1+i)^t} + \frac{B}{(1+i)^n}$$

where

P_0 is the intrinsic value of the bond,
cB is the periodic coupon interest paid at the end of period,
c is a coupon rate,
B is the face value,
i is yield to maturity (YTM),
n is the number of periods that remain before the bond is redeemed,
B is the principal (or face amount) of the bond.

The following bond valuation formula assumes annual compounding.

(1) $$P = \frac{cB}{(1+i)^1} + \frac{cB}{(1+i)^2} + \ldots + \frac{cB+B}{(1+i)^T}$$

or

(2) $$P = \frac{B\left(c\left[(1+i)^T - 1\right] + i\right)}{i(1+i)^T}$$

Semiannual Payments

If instead of annual payments, the bond pays interest semiannually, the value of a bond is

(4) $$P_0 = \sum_{t=1}^{2n} \frac{\frac{cB}{2}}{\left(1+\frac{y}{2}\right)^t} + \frac{B}{\left(1+\frac{y}{2}\right)^{2n}}$$

Whenever a bond's RRR is greater than the coupon rate, it sells at a discount. When the coupon rate exceeds the RRR, it sells at a premium.

With semiannual compounding the bond price is equal to:

(3) $$P = \frac{\frac{c}{2}B}{\left(1+\frac{y}{2}\right)^1} + \frac{\frac{c}{2}B}{\left(1+\frac{y}{2}\right)^2} + \ldots + \frac{\frac{c}{2}B + B}{\left(1+\frac{y}{2}\right)^T}$$

or

(4) $$P = \frac{B\left(c\left[\left(1+\frac{y}{2}\right)^T - 1\right] + y\right)}{y\left(1+\frac{y}{2}\right)^T}$$

Bond with m Coupons in a Year

With m coupons in a year a bond price is equal to

(5) $$P = \frac{\frac{c}{m}B}{\left(1+\frac{y}{m}\right)^1} + \frac{\frac{c}{m}B}{\left(1+\frac{y}{m}\right)^2} + \ldots + \frac{\frac{c}{m}B}{\left(1+\frac{y}{m}\right)^{nm}} + \frac{B}{\left(1+\frac{y}{m}\right)^{nm}}$$

or

(6) $$P = cB\left[\frac{\left(1+\frac{y}{m}\right)^{nm} - 1}{y\left(1+\frac{y}{m}\right)^{nm}}\right] + \frac{B}{\left(1+\frac{y}{m}\right)^{nm}}$$

or

(7) $$P = B\frac{c\left[\left(1+\frac{y}{m}\right)^{nm} - 1\right] + y}{y\left(1+\frac{y}{m}\right)^{nm}}$$

The relationship between a bond's yield y with m-coupons and an effective yield i (bond with annual cash flows) is

$$(1+i)^n = \left(1+\frac{y}{m}\right)^{mn}$$

and

$$y = m\left[(1+i)^{\frac{1}{m}} - 1\right]$$

Thus the price of a bond with m coupons is:

(8) $$P = B\frac{c\left[(1+i)^n - 1\right] + m\left[(1+i)^{\frac{1}{m}} - 1\right]}{m\left[(1+i)^{\frac{1}{m}} - 1\right](1+i)^n}$$

Zero Coupon Bonds

The price of a zero coupon bond is

(9) $$P = \frac{CF_T}{(1+i)^T} = \frac{B}{(1+i)^T}$$

Yield to maturity (internal rate of return) is:

(10) $$YTM = \left(\frac{B}{P_M}\right)^{\frac{1}{T}} - 1$$

Console Bonds

A console bond never matures. It is a perpetuity. There is no maturity. Note, that while maturity is infinite ∞), duration is finite.

(11) $$P = \sum_{t=1}^{\infty}\frac{CF_t}{(1+i)^t} = \frac{Bc}{i}$$

With semiannual compounding the consol price is equal to:

$$(12) \quad P = \frac{B\frac{c}{2}}{\frac{y}{2}}$$

Discount Rate

Some money market instruments (Treasury bills, NBP bills) are quoted on discount rate basis. The discount rate is calculated as the discount divided by the face value of the bill multiplied by the number of periods of length t in a 360-day year:

$$d_t = \frac{\text{discount}}{\text{face value}} \frac{360}{t}$$

Actually in Poland the appropriate yield for bills is calculated (spot rate) on a a/360 days basis.

$$z_t = \frac{\text{discount}}{\text{face value - discount}} \frac{360}{t} = \left(\frac{\text{face value}}{\text{face value - discount}} - 1 \right) \frac{360}{t}$$

It can be shown that two rates are interrelated:

$$d_t = \frac{z_t}{1 + z_t \frac{t}{360}} \quad \text{and} \quad z_t = \frac{d_t}{1 - d_t \frac{t}{360}}$$

Because yield for bills is calculated on a/360 basis, we can calculate the equivalent money market yield:

$$y_t = z_t \frac{365}{360}$$

All the above stated three rates are not annualized returns. The effective annualized rate is

$$i = \left(1 + y_t \frac{t}{365} \right)^{\frac{365}{t}} - 1 \quad \text{or} \quad i = \left(1 + z_t \frac{t}{360} \right)^{\frac{365}{t}} - 1$$

Problem 2. Discount rate and effective rate

The face value of a Treasury bill is 10000 zł. Maturity is 65 days. The yield is 4,50%.
(a) What is the present value of one bill ?
(b) Calculate discount, discount rate, spot rate (a/365) and effective annualized rate.
(c) What is your net income after tax (tax rate is 20 per cent).

Solution

(a)

Price 9919,40

Yield (10000/9919,40 - 1)* 360/65=4,500%

(b)

Discount	80,60 zł
Discount rate	4,464%
Spot rate a/360 (z_t)	4,500%
Spot rate a/365 (y_t)	4,562%
Effective rate (i)	4,649%

$$i = \left(1 + y_t \frac{t}{365}\right)^{\frac{365}{t}} - 1$$

(c)

Interest after tax is equal to 80,60 x 0,8 =64,48.

Problem 3. Discount rate and effective rate

Treasury bills auction on October 2, 2003

Value date	03-10-03
Type of bill	10 tyg.
ISIN	PL0000000790
Maturity	12-12-03
Supply	3500,00 mln zł
Demand	6243,20 mln zł
Accepted offers	3500,00 mln zł
Minimum price	9 899,01 zł
Average price	9 899,77 zł
Maximum price	9 903,71 zł
Maximum yield	5,247%
Average yield	5,207%
Minimum yield	5,000%

(a) Show calculations of maximum, minimum and yield.

(b) Calculate, discount, discounting, rate and effective annualized rate.

Solution

(a)

Number of days to maturity		70
Maximum yield	(10000/9899,01 - 1)* 360/70=5,247%	
Average yield	(10000/9899,77 - 1)* 360/70=5,207%	
Minimum yield	(10000/9903,71 - 1)* 360/70=5,000%	

(b)

Discount	Discount rate	Spot rate	Effective rate
100,99 zł	5,194%	5,247%	5,359%
100,23 zł	5,155%	5,207%	5,318%
96,29 zł	4,952%	5,000%	5,102%

Continuously Compounded Rate

(13) $c = \ln(1+i)$

1.1.1 The term structure of interest rates

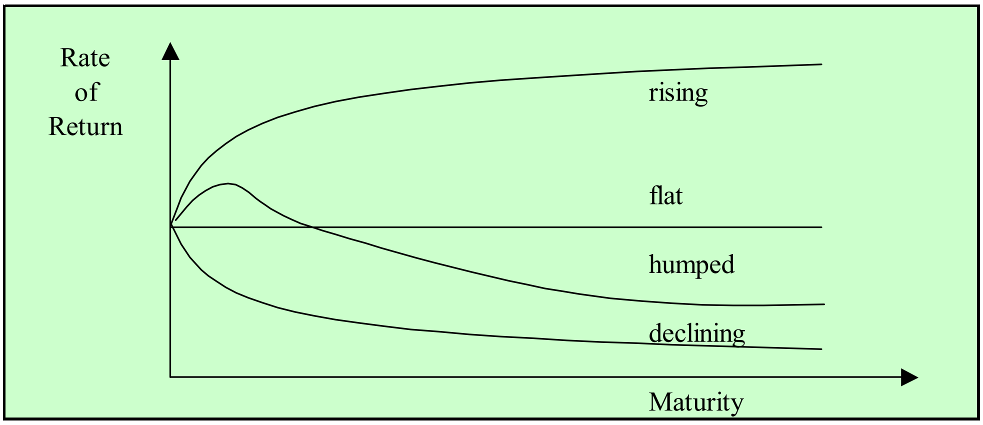
The term structure of interest rates is a function that relates the term to maturity usually to annualized interest rates.

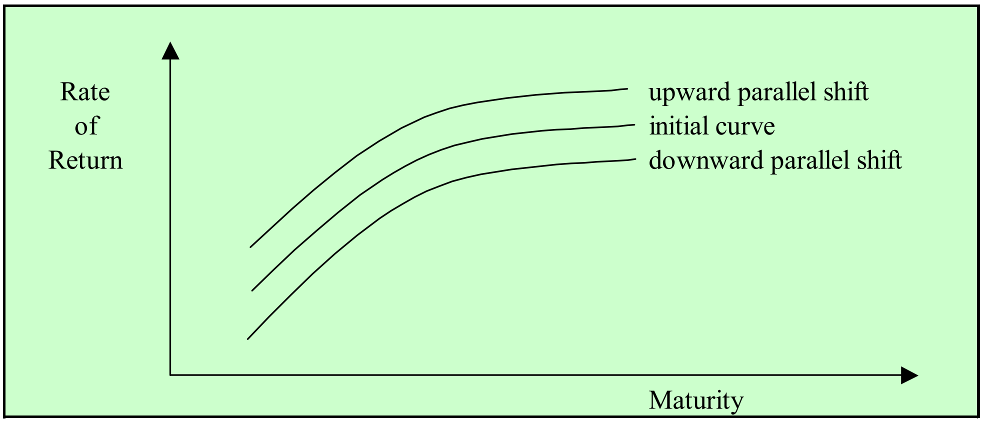
A yield curve is a diagrammatic representation of the term structure of annualized interest rates. This can be done for any interest bearing instrument. A graph can be drawn of the yield for each asset against the number of years to maturity; the best curve through this set of points is called the yield curve. Term structures of YTM (yield to maturity, gross redemption rate) for riskless government securities are often published in newspapers or on Web sites.

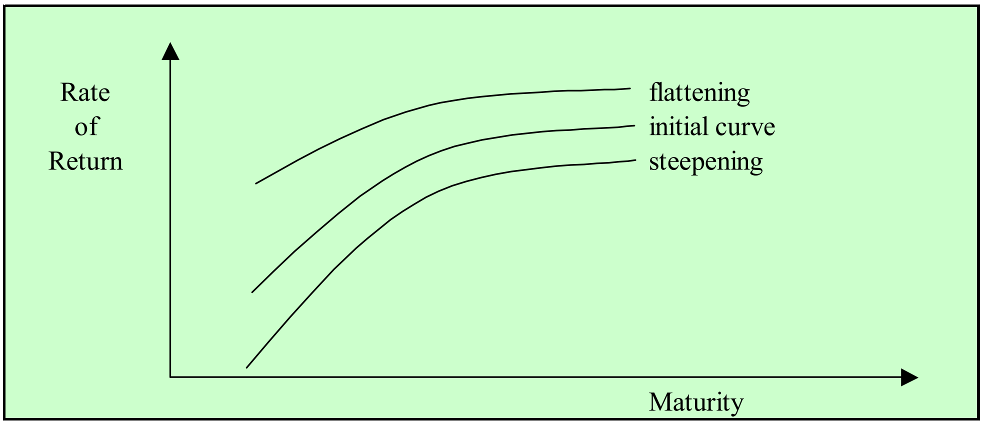
In the typical situation, yields are rising as the term to maturity increases. Sometimes yield curve is falling yield curve (it is called an inverted curve). A flat curve does not occur in practice, but sometimes the assumption of flat curve is used in theoretical models.

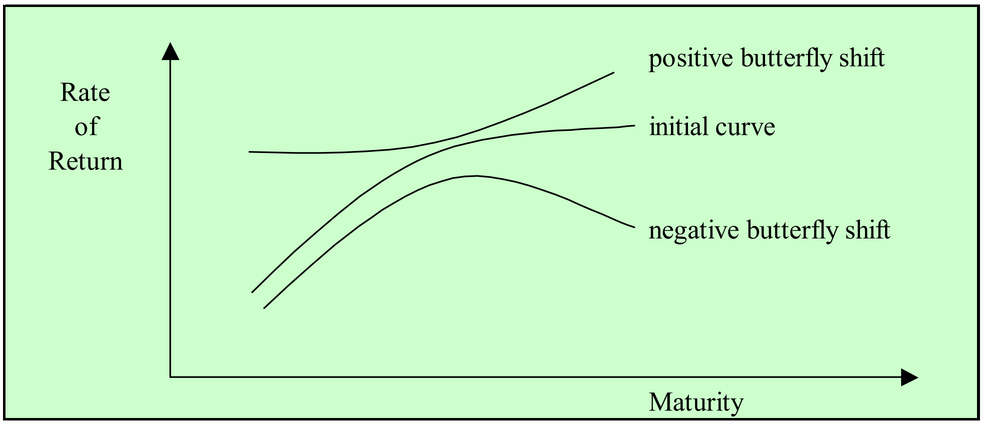
The shape of yield curve is explained by number of theories. The most popular are:

- expectations theory,
- liquidity preference theory,
- market segmentation theory.









According to the expectation theory rising curve indicates that rates of interest are expected to rise in future. A falling yield curve implies that interest rates are expected to fall.

The normal shape of the curve (upwards sloping) can be explained by liquidity preference theory. Investors need to be compensated with a higher yield for being deprived of their cash for a longer period of time.

Market segmentation theory states that there are different categories of investors who are interested in different segments of the curve. Typically, banks invest at the short end of the market while pension funds, insurance companies and some investment funds buy and sell long-term securities.

Financial managers should inspect the current shape of the yield curve when deciding on investments or borrowings. For example, a yield curve sloping steeply upwards suggests that interest rates will rise in the future. The manager may therefore wish to avoid borrowing long-term on variable rates, since the interest charge may increase considerably over the term of the loan. Short-term variable rate borrowing or long-term fixed rate borrowing may instead be more appropriate.

The term structure of interest rates depends on monetary policy.

1.1.2 Factors influencing interest rates

The main factors affecting the rate appropriate to a particular type of financial asset are as follows.

Maturity (duration)

Short-term interest rates are normally lower than longer-term rates of interest. The simple reason for this is that the longer the period of a loan the more the risk for the lender. Uncertainty is greater and the possibility of default increases, hence an investor will want a higher rate of return to compensate him for this enhanced degree of risk on a longer-term loan.

Short-term and long-term usually often up and down together, but short-term interest are much more volatile. It is possible for short-term interest rates to be temporarily higher than longer-term rates, e.g. as the cause or result of a foreign exchange crisis.

Credit risk

Higher-risk borrowers will have to pay higher yields on their borrowing, to compensate lenders for the greater risk involved.

Banks assess the creditworthiness of the borrower, and set a rate of interest on its loan at a certain mark-up above its base rate or WIBOR. In general, larger companies are charged at a lower rate of interest than smaller companies.

Size of the loan or deposit

The interest rate might vary with the size of the loan or deposit. The deposits above a certain amount will probably attract higher rates of interest than smaller-sized time deposits. The administrative costs of handling wholesale loans rather than a large number of small retail loans partially explains the lower rates of interest charged by banks on larger loans.