2. Measuring Risk. Traditional Measures. Value at Risk. EaR. CFaR. Hedging and Speculation

Risk arises from the variability of future returns, values, cash flows, earnings and other stated goals. This variability is caused by changes in prices (price risk), exchange rates (currency risk), interest rates (interest rate risk), creditworthiness (credit risk) and many other factors. Price risk, currency risk, interest rate risk and credit risk may be hedged with derivatives.

There are many risk measures:

- traditional (variance, standard deviation, coefficient of variation),
- modern (Value at Risk, Cash flows at Risk, Earnings at Risk),
- specific (duration gap, currency gap).

Traditional risk measures

The most popular measure of risk is the variance or the standard deviation of a distribution. The **variance** (σ^2) of returns of a two-asset portfolio is calculated as

$$\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho_{AB}\sigma_A\sigma_B$$

where:
w_i - the percent of the portfolio in asset i,
σ_i - standard deviation of return i,
ρ_{ij} - correlation coefficient of returns.

The **standard deviation** (σ) of returns is the square root of the variance of the distribution.

$$\sigma = \sqrt{\sigma^2}$$

It is a statistical measure of the dispersion of returns around the expected value. A larger standard deviation indicates greater dispersion. It is a common measure of volatility.

The **correlation coefficient** between stocks A and B is denoted by ρ_{AB} and can be calculated from the covariance:

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \times \sigma_B}$$

The correlation coefficient, being a standardized value, always lies between -1 and 1 (both values inclusive). If the correlation coefficient between two securities is +1, the two securities are said to be perfectly positively correlated. On the other hand, a correlation coefficient of -1 implies that the two assets are perfectly negatively correlated. Correlation should not be confused with causation.

The combination of two assets that are completely negatively correlated provides the maximum benefits of diversification.

Coefficient of variation is a relative measure of risk (standard deviation of returns/expected rate of return).

Problem 4

Two companies respond to the economy in the following manner:

Event	*Probability*	*Return on ABC Co. Shares*	*Return on XYZ Co. Shares*
Economic upturn	0,30	0,12	0,24
No changes in the economy	0,40	0,18	0,18
Economic downturn	0,30	0,24	0,12

It is argued that because both shares have the same expected return and the same risk (as measured by standard deviation of returns), investors will be indifferent to buying shares of either of the two companies. Is this true ?

The variance of returns of n-assets portfolio is:

$$\sigma_P^2 = \mathbf{w}^T\mathbf{V}\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22}^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn}^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

or (more practical approach)

$$\sigma_P^2 = u^T R u = \begin{bmatrix} w_1\sigma_1 & w_2\sigma_2 & \dots & w_n\sigma_n \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} w_1\sigma_1 \\ w_2\sigma_2 \\ \vdots \\ w_n\sigma_n \end{bmatrix}$$

Systematic risk and unique risk

As the size of a portfolio increases, the portfolio's variance is dependent more on the covariances among the securities in the portfolio than on variances of individual securities. As the number of securities in the portfolio increases, the risk is reduced. It is convenient to divide total risk (variance risk) into two distinct components: **undiversifiable risks** (the covariance risk) and **diversifiable risk** (the remaining risk in the portfolio). Undiversifiable risks are **market risks**, also known as **systematic risks** (beta risks). **Unique** (diversifiable) risks are risks that are specific to a company. These are risks that can be diversified away by forming a large portfolio.

Beta Coefficient

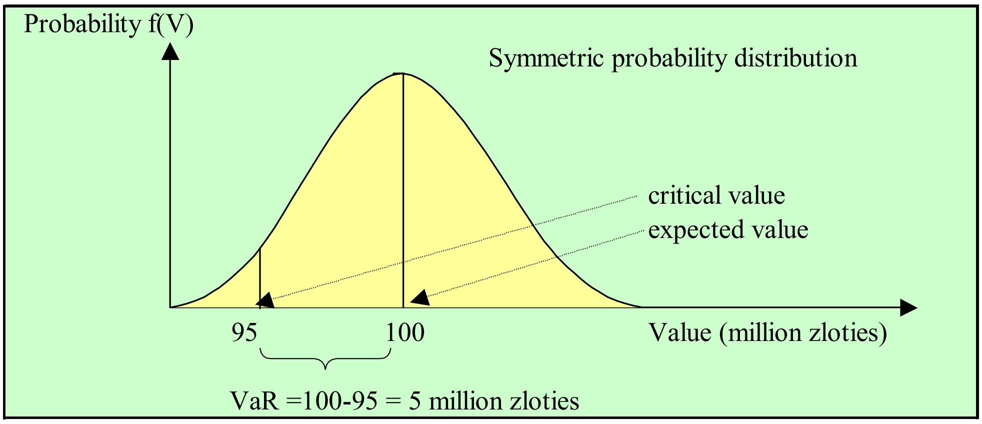
An asset's systematic risk depend on the **beta** of the asset, which measures the **nondiversifiable risk** of an asset, thus relating the asset's risk to the risk of the market. The **beta coefficient** for asset i is given by

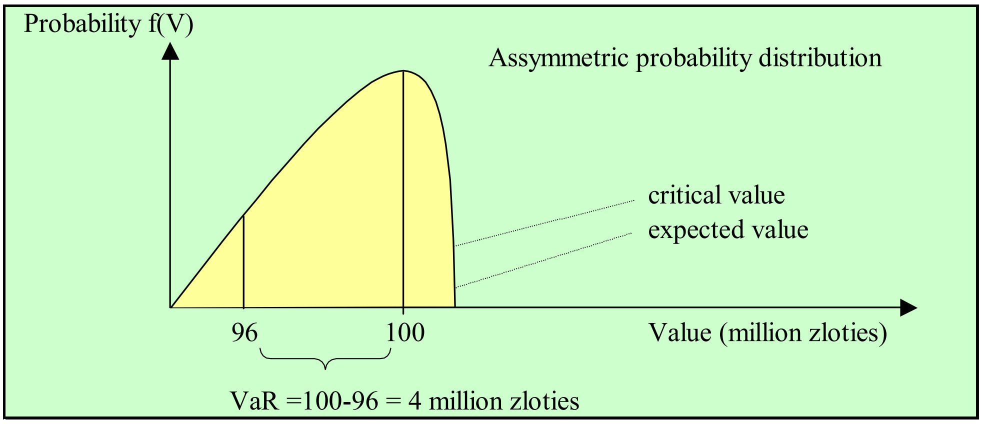
$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \rho_{im}\frac{\sigma_i}{\sigma_m}$$

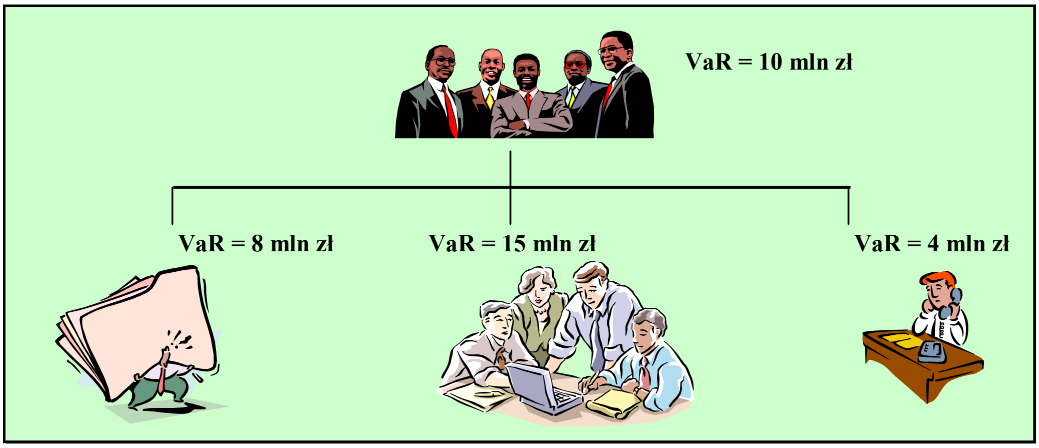
Beta is a statistical concept that relates the sensitivity of a security's returns to changes in the returns of the market. Beta measures how responsive an asset is to market movements. A beta of 1.0 indicates an asset of average risk. A stock with a beta greater than 1.0 is an above-average-risk stock, and its returns are more volatile than those of the market.

VaR, CfaR, EaR

Value-at-Risk is a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a pre-set horizon.







There are many methods to calculate VaR. The most popular are: historical simulation method, variance/covariance method and Monte Carlo simulation method.

Historical Simulation Method. Historical simulation method is used to find an empirical distribution of the rates of return assuming that past history carries out into the future. Historical simulation is a simple, theoretical approach that requires relatively few assumptions about the statistical distributions of the underlying market factors. In essence, the approach involves using historical changes in market rates and prices to construct a distribution of potential future portfolio profits and losses, and then reading off the value at risk as the loss that is exceeded only 5% of the time.

Correlation Method. The correlation method, otherwise known as the variance/covariance method, is essentially a parametric approach in which an estimate of VAR is derived from the underlying variances and covariances of the constituents of a portfolio. The variance/covariance approach is based on the assumption that the underlying market factors have a multivariate normal distribution. If a probability of 5 percent is used in determining the value at risk, then the value at risk is equal to 1.65 times the standard deviation of changes in portfolio value:

$$\mathrm{VaR} \cong 1{,}65\sigma_P V_{t-1}$$

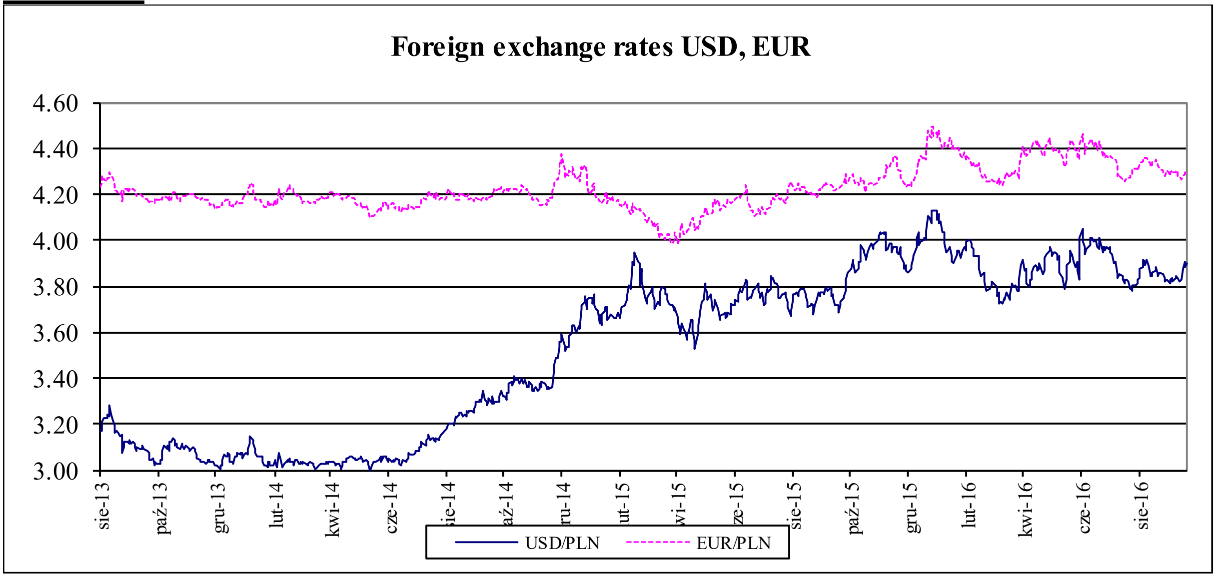
Monte Carlo Simulation. The Monte Carlo simulation methodology has a number of similarities to historical simulation. The main difference is that rather than carrying out the simulation using the observed changes in the market factors over the historical periods to generate hypothetical portfolio profits or losses, one chooses a statistical distribution that is believed to adequately capture or approximate the possible changes in the market factors. Then, a pseudo-random number generator is used to generate thousands of hypothetical scenarios of changes in the market factors. These are then used to construct the distribution of possible portfolio profit or loss. Finally, the value at risk is then determined from this distribution.

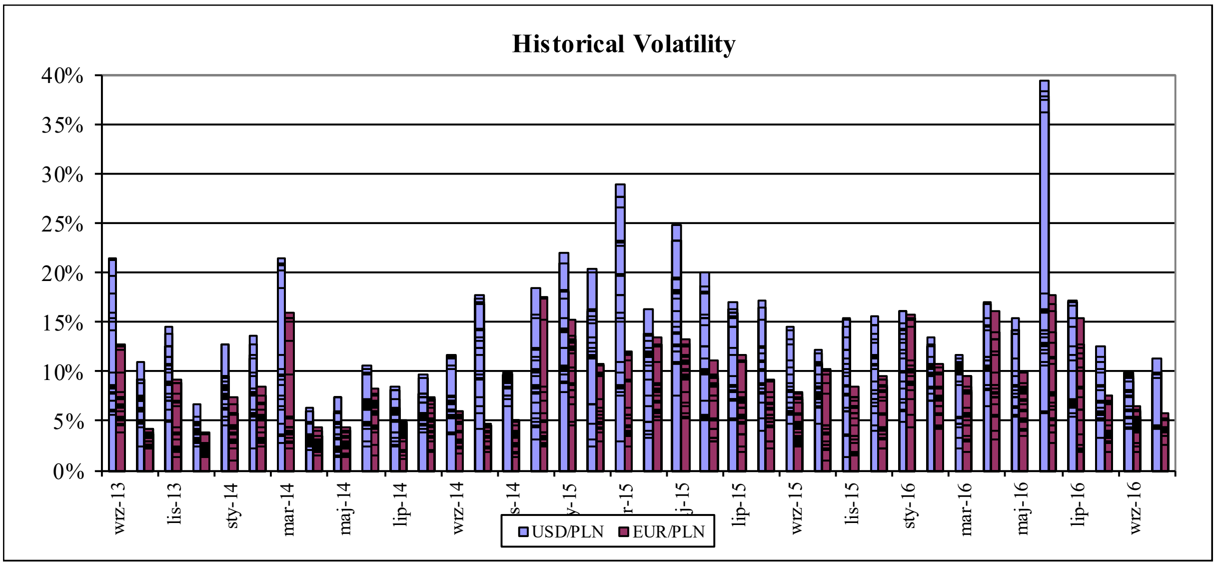
The similar risk measures are CFaR and EaR.

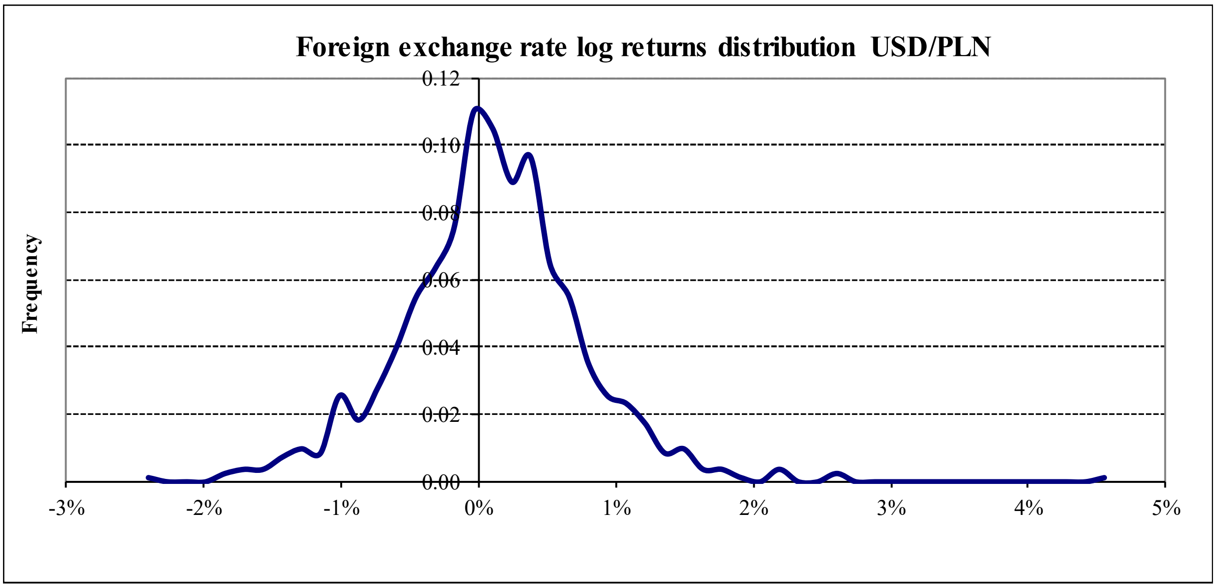
Cash-Flow-at-Risk (CFaR). The maximum shortfall of net cash generated, relative to a specified target, that could be experienced due to the impact of market risk on a specified set of exposures, for a specified reporting period and confidence level.

Earnings at Risk (EaR) The maximum shortfall of earnings, relative to a specified target, that could be experienced due to the impact of market risk on a specified set of exposures, for a specified reporting period and confidence level.

Problem 5


Foreign exchange rates USD, EUR
USD/PLN
EUR/PLN


Historical Volatility
USD/PLN
EUR/PLN


Foreign exchange rate log returns distribution USD/PLN
Frequency

Historical standard deviation of foreign exchange rates returns for a period 26-08-2013 - 16-09-2016

	USD	EUR
Standard deviation	0.667%	0.405%
Correlation matrix	100.00%	53.36%
	53.36%	100.00%

Portfolio variance computed using correlation matrix

	USD	EUR
Portfolio weights	60%	40%
Portfolio variance	0.003%	
Standard deviation	0.506%	

Value at Risk

a/2	$t_{a/2}$	s_{rH}	V_{t-1}	VaR
5.00%	1.645	0.506%	100	0.832
2.00%	2.054	0.506%	100	1.038
1.00%	2.326	0.506%	100	1.176

Specific risk measures

The specific risk measures are the most important. In general an "open gap" means that specific factors (exchange rates, interest rates) may influence the value, cash flows, earnings, returns.

Duration gap

Duration is a measure of the interest rate sensitivity of a security, but also of assets, liabilities or equity capital. The durations of the assets and liabilities are obtained as weighted averages. Positive balance duration gap means a company is exposed to rising interest rates. A negative gap value, which would occur when short-term assets are financed with longer term liabilities, creates a negative duration gap. Negative duration gap means a company is exposed to falling interest rates.
Positive currency gap means a company is exposed to falling exchange rates. A negative currency gap means a company is exposed to rising exchange rates.

$$\Delta W = -D\Delta i W$$

$$\Delta E = -D \cdot A \cdot \Delta y$$

$$\Delta W = \left[-D\Delta i + \frac{1}{2} C(\Delta i)^2 \right] W$$

Currency gap

Positive currency gap means a company is exposed to falling exchange rates. A negative currency gap means a company is exposed to rising exchange rates.

Greeks (options)

Measure	Notation	Call	Put
Delta	$\Delta_c = \frac{\partial C}{\partial S}$	$\Delta_c = N(d_1)$	$\Delta_p = \Delta_c - 1$
Gamma	$\gamma_c = \frac{\partial^2 C}{\partial S^2}$	$\gamma_c = \frac{n(d_1)}{S\sigma\sqrt{T}}$	$\gamma_p = \gamma_c$
Theta	$\theta_c = -\frac{\partial C}{\partial T}$	$\theta_c = \frac{-S\sigma n(d_1)}{2\sqrt{T}} - rEe^{-rT}N(d_2)$	$\theta_p = \theta_c + rEe^{-rT}$
Rho	$\rho_c = \frac{\partial C}{\partial r}$	$\rho_c = TEe^{-rT}N(d_2)$	$\rho_p = \rho_c - TEe^{-rT}$
Vega	$\upsilon_c = \frac{\partial C}{\partial \sigma}$	$\upsilon_c = S\sqrt{T}n(d_1)$	$\upsilon_p = \upsilon_c$

where: $n(d) = \frac{e^{-d^2/2}}{\sqrt{2\pi}}$