1. Currency Risk Management with Forward, Futures and Options. Hedge Ratios. Currency Strategies. Performance Attribution

1.1 Unhedged and Hedged Position

Forward Surprise

Foreign currency risk premium is the difference between the percentage change in the spot exchange rate and the forward premium (or discount)

(1) $$r_c = r_d - f = \frac{S_t - F}{S_0}$$

This premium is also referred as the forward contract return or the forward surprise.

Unhedged position

If the currency exposure is unhedged the discrete return on a foreign asset translated into home currency units can be written as

(2) $$r_N = (1 + r_z)(1 + r_d) - 1$$

where

r_z – the foreign asset return,

r_d – the percentage change in the spot exchange rate using the home currency/foreign currency convention.

The approximate unhedged return to the domestic investor is

(3) $$r_N \approx r_z + r_d$$

The continuously compounded return would be

(4) $$\ln(1 + r_N) = \ln(1 + r_z) + \ln(1 + r_d) \qquad \text{or} \qquad r_N^* = r_z^* + r_d^*$$

Consider an investment in US T-bills with one year maturity and yield to maturity equal to $r_z = i_f^N$. Forward premium is approximately equal to the difference between domestic risk-free rate and foreign risk-free rate.

(5) $$f \approx \left(i_d^N - i_f^N\right)$$

The approximate unhedged return to the domestic investor is

(6) $$r_N \approx i_f^N + r_d = i_d^N + r_d - f = i_d^N + r_c$$

The unhedged return is equal to

- foreign risk-free rate plus currency return (percentage change in the exchange rate), or
- domestic risk-free rate plus the foreign currency risk premium (forward surprise).

Hedged position

The currency exposure in a foreign asset may be hedged by selling a forward contract. The hedged return on the foreign asset will be equal to:

(7) $r_H = r_N + h(r_d - f)$

where
r_N – unhedgd return,
h – hedge ratio,
r_d - f – forward contract return.

The hedge ratio (h) is the coefficient indicating what portion of the total exposure should be hedged. The negative hedge ratio (h) refers to short (selling) position, while the positive hedge ratio is used to indicate long (buying) position in the derivative instrument (forward or futures contract). A long currency position or positive currency gap may be hedged by selling forward or futures contracts (negative hedge ratio). A short currency position may be hedged by buying forward or futures contracts (positive hedge ratio). Such convention is also very useful when a currency portfolio consists of many assets and derivatives including currency options. The sign of the hedge ratio indicates what an investor should do (sell or buy).

To hedge the principal and the interest, the hedge ratio should be equal to h = -(1+r_z*T). When T = 1 the fully hedged foreign asset return is approximately equal to asset return plus forward premium. The exact return is equal to

(8) $r_H = (1 + r_z)(1 + r_d) - 1 - (1 + r_z)(r_d - f) = (1 + r_z)(1 + f) - 1$

When we add arbitrage relationship (covered interest arbitrage) the hedged return is

(9) $r_H = (1 + r_z)(1 + f) - 1 = (1 + r_z)\frac{F}{S_0} - 1 = (1 + r_z)\frac{\left(1 + i_d^N\right)}{\left(1 + i_f^N\right)} - 1$

In addition we may assume that the long currency position consists of risk-free rate foreign government securities ($r_z = i_f^N$), and we have

(10) $r_H = i_d^N$

The fully hedged return on the risk-free investment abroad is equal to the domestic risk-free rate.

The hedged return on the long currency position hedged by selling a forward contract is

(11) $r_H = (1 + r_z)(1 + r_d) - 1 + h(r_d - f) \approx r_z + r_d + h(r_d - f)$

where
r_z – the foreign asset return,
r_d – the percentage change in the spot exchange rate using the home currency/foreign currency convention,
h – hedge ratio,

f – forward premium.

The hedge ratio h may be replaced with the currency exposure ratio H (H=1+h). The last indicates the proportion of the total currency asset exposure left unhedged. When the currency exposure is unhedged H=1, when the currency exposure is fully hedged H=0

With the currency exposure ratio the hedged return is equal to:

(12) $$r_H \approx r_z + r_d + (H-1)(r_d - f) = r_z + f + H(r_d - f)$$

When H=1 (unhedged position) return is equal to the asset return plus the unknown future currency return

(13) $$r_N \approx r_z + r_d$$

When H=0 (fully hedged position) return is equal to the asset return plus the forward premium (both of them are known are known in advance)

(14) $$r_H \approx r_z + f$$

The relationship between the unhedged return and the hedged return are presented in the following figure

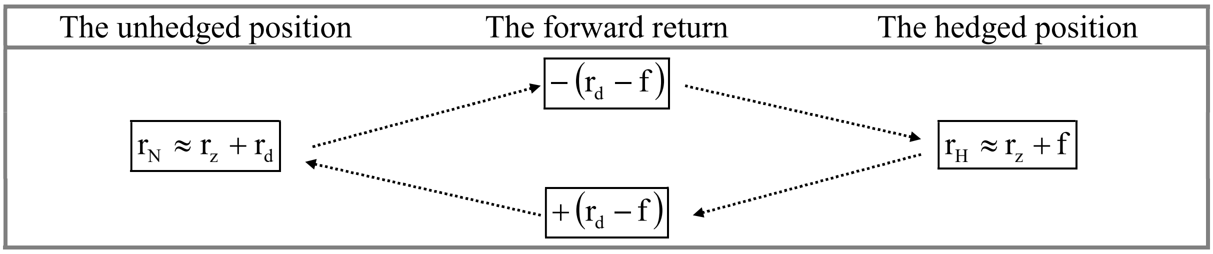

Figure 1. The unhedged and hedged currency position

Optimization of return on hedged currency position

The optimization model may have the following objective function

(15) $$E(r_z) + f + H[E(r_d) - f] - \alpha\left(\sigma_{r_z}^2 + H^2\sigma_{r_d}^2 + 2H\rho_{r_z r_d}\sigma_{r_z}\sigma_{r_d}\right)$$

where

$E(r_z)$- expected foreign asset return,

f – the currency forward premium or discount,

$E(r_d) - f$ expected return of the forward contract („forward surprise”),

$E(r_d)$- expected currency return,

H – currency exposure ratio,

α - risk aversion penalty,

$\sigma_{r_z}^2$ - variance of foreign asset return,

$\sigma_{r_d}^2$ - variance of the currency return,

$\rho_{r_z r_d}\sigma_{r_z}\sigma_{r_d}$ - covariance between the foreign asset return and the currency return,

$\rho_{r_z r_d}$ - correlation coefficient between the foreign asset return and the currency return.

The objective function maximizes risk adjusted currency portfolio return. Currency risk is measured by the variance of portfolio return.

Problem 1. The unhedged and hedged position with forward, options and FX swap

A bank invests $10 million in the US T-bills with 1 year maturity. Yield to maturity is 5%.
The spot exchange rate is 4,00 PLN/USD. The domestic risk-free rate is 12%.
The foreign risk-free rate is 5%.
(a) Calculate the unhedged return when foreign exchange rate is changed by +- 10%, 20%, 30% ?
(b) Calculate the hedged return when the position is hedged with a forward contract.
Calculate the forward exchange rate.
What are the profits/losses against the unhedged position?
Compare the the hedged return with a domestic risk-free return.
(c) Calculate the hedged return for a covered call and protective put strategies.
The exercise exchange rate is 4,00 PLN/USD, standard deviation is 10%.
Option prices are established using BSM model.
(d) Show the currency hedging using FX swap with a 1 year maturity.
What capital transactions may replicate the FX swap ?

Solution

(a)

Situation in t=0

Exposure in foreign currency	9,524	million USD
Spot exchange rate	4,00	PLN/USD
Exposure in domestic currency	38,095	million PLN

Unhedged position in t=1

Curency return r_d	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
Exposure in foreign currency	10,000	10,000	10,000	10,000	10,000	10,000	10,000
Spot exchange rate (t=1)	2,80	3,20	3,60	4,00	4,40	4,80	5,20
Value of the unhedged position	28,000	32,000	36,000	40,000	44,000	48,000	52,000
The unhedged return	-26,5%	-16,0%	-5,5%	5,0%	15,5%	26,0%	36,5%
Return on foreign asset r_z	5,0%	5,0%	5,0%	5,0%	5,0%	5,0%	5,0%
Currency return r_d	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
The unhedged return $(1+r_z)(1+r_d)-1$	-26,5%	-16,0%	-5,5%	5,0%	15,5%	26,0%	36,5%
The approximate unhedged return $= r_z+r_d$	-25,0%	-15,0%	-5,0%	5,0%	15,0%	25,0%	35,0%

(b)

Hedged position with a forward contract

Forward exchange rate 4,2667

Time t=1

Curency return r_d	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
Exposure in foreign currency	10,000	10,000	10,000	10,000	10,000	10,000	10,000
Forward exchange rate	4,27	4,27	4,27	4,27	4,27	4,27	4,27
Value in domestic currency	42,667	42,667	42,667	42,667	42,667	42,667	42,667
The hedged return in domestic currency	12,0%	12,0%	12,0%	12,0%	12,0%	12,0%	12,0%
Gains / losses against unhedged position	14,667	10,667	6,667	2,667	-1,333	-5,333	-9,333
Currency hedge return	38,5%	28,0%	17,5%	7,0%	-3,5%	-14,0%	-24,5%
Currency return $r_d = S/S_0 - 1$	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
- forward premium $f = F/S_0 - 1$	6,7%	6,7%	6,7%	6,7%	6,7%	6,7%	6,7%
Forward contract return (forward surprise) $(S-F)/S_0 = r_d-f$	-36,7%	-26,7%	-16,7%	-6,7%	3,3%	13,3%	23,3%
Hedge ratio h	-1,05	-1,05	-1,05	-1,05	-1,05	-1,05	-1,05
Currency hedge return $h(r_d-f)$	38,5%	28,0%	17,5%	7,0%	-3,5%	-14,0%	-24,5%
+ the unhedged return $(1+r_z)(1+r_d)-1$	-26,5%	-16,0%	-5,5%	5,0%	15,5%	26,0%	36,5%
The hedged return on the foreign asset $(1+r_z)(1+r_d)-h^*(r_d-f)$	12,0%	12,0%	12,0%	12,0%	12,0%	12,0%	12,0%
Approximation							
Asset return + forward premium r_z+f	11,7%	11,7%	11,7%	11,7%	11,7%	11,7%	11,7%
Currency exposure ratio H=h+1	-5,0%	-5,0%	-5,0%	-5,0%	-5,0%	-5,0%	-5,0%
$H^*(r_d-f)$	1,8%	1,3%	0,8%	0,3%	-0,2%	-0,7%	-1,2%
The appoximated hedged return on foreign asset $r_z+f+H(r_d-f)$	13,5%	13,0%	12,5%	12,0%	11,5%	11,0%	10,5%
Approximation error $r_z r_d$	-1,5%	-1,0%	-0,5%	0,0%	0,5%	1,0%	1,5%
Total gains							
ΔS	-10,095	-6,095	-2,095	1,905	5,905	9,905	13,905
ΔF	-13,968	-10,159	-6,349	-2,540	1,270	5,079	8,889
$\Delta V=\Delta S+h\Delta F$	4,571	4,571	4,571	4,571	4,571	4,571	4,571

(c)

Hedging - Short CALL

Premium is calculated using BSM model 0,3097 PLN/USD

Sensitivity measures

Delta	Gamma	Theta	Rho	Vega
0,77	0,79	0,00	0,03	0,01

Equal-dollar hedge ratio -1

Curency return r_d	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
Profit - premium received	3,097	3,097	3,097	3,097	3,097	3,097	3,097
Loss, when the foreign exchange rate in time t=1 is above the strike price	0,000	0,000	0,000	0,000	-4,000	-8,000	-12,000
Profit / Loss from Short CALL	3,097	3,097	3,097	3,097	-0,903	-4,903	-8,903
Value of the unhedged position	28,000	32,000	36,000	40,000	44,000	48,000	52,000
Value of the hedged position	31,097	35,097	39,097	43,097	43,097	43,097	43,097
The unhedged return	-18,4%	-7,9%	2,6%	13,1%	13,1%	13,1%	13,1%
Total gains							
ΔS	-10,095	-6,095	-2,095	1,905	5,905	9,905	13,905
ΔC	-3,097	-3,097	-3,097	-3,097	0,903	4,903	8,903
$\Delta V=\Delta S+h\Delta C$	-6,998	-2,998	1,002	5,002	5,002	5,002	5,002

Delta-neutral hedge ratio $h=-\Delta I/\Delta_c$ -1,293

Curency return r_d	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
Profit - premium received	4,004	4,004	4,004	4,004	4,004	4,004	4,004
Loss, when the foreign exchange rate in time t=1 is above the strike price	0,000	0,000	0,000	0,000	-5,172	-10,344	-15,516
Profit / Loss from Short CALL	4,004	4,004	4,004	4,004	-1,168	-6,340	-11,512
Value of the unhedged position	28,000	32,000	36,000	40,000	44,000	48,000	52,000
Value of the hedged position	32,004	36,004	40,004	44,004	42,832	41,660	40,488
The unhedged return	-16,0%	-5,5%	5,0%	15,5%	12,4%	9,4%	6,3%
Total gains							
ΔS	-10,095	-6,095	-2,095	1,905	5,905	9,905	13,905
ΔC	-3,097	-3,097	-3,097	-3,097	0,903	4,903	8,903
$\Delta V=\Delta S+h\Delta C$	-6,091	-2,091	1,909	5,909	4,737	3,565	2,393

Hedging - Long PUT

Premium is calculated using BSM model 0,0525 PLN/USD

Sensitivity measures

Delta	Gamma	Theta	Rho	Vega
-0,23	0,79	0,00	-0,01	0,01

Equal-dollar hedge ratio 1

Curency return r_d	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
Loss - premium paid	-0,525	-0,525	-0,525	-0,525	-0,525	-0,525	-0,525
Loss, when the foreign exchange rate in time t=1 is below the strike price	12,000	8,000	4,000	0,000	0,000	0,000	0,000
Profit / Loss from Long Put	11,475	7,475	3,475	-0,525	-0,525	-0,525	-0,525
Value of the unhedged position	28,000	32,000	36,000	40,000	44,000	48,000	52,000
Value of the hedged position	39,475	39,475	39,475	39,475	43,475	47,475	51,475
The unhedged return	3,6%	3,6%	3,6%	3,6%	14,1%	24,6%	35,1%
Total gains							
ΔS	-10,095	-6,095	-2,095	1,905	5,905	9,905	13,905
ΔP	11,475	7,475	3,475	-0,525	-0,525	-0,525	-0,525
$\Delta V=\Delta S+h\Delta P$	1,380	1,380	1,380	1,380	5,380	9,380	13,380

Delta-neutral hedge ratio $h=-\Delta I/\Delta_p$ 4,413

Curency return r_d	-30,0%	-20,0%	-10,0%	0,0%	10,0%	20,0%	30,0%
Loss - premium paid	-2,315	-2,315	-2,315	-2,315	-2,315	-2,315	-2,315
Loss, when the foreign exchange rate in time t=1 is below the strike price	52,950	35,300	17,650	0,000	0,000	0,000	0,000
Profit / Loss from Long Put	50,636	32,986	15,335	-2,315	-2,315	-2,315	-2,315
Value of the unhedged position	28,000	32,000	36,000	40,000	44,000	48,000	52,000
Value of the hedged position	78,636	64,986	51,335	37,685	41,685	45,685	49,685
The unhedged return	106,4%	70,6%	34,8%	-1,1%	9,4%	19,9%	30,4%
Total gains							
ΔS	-10,095	-6,095	-2,095	1,905	5,905	9,905	13,905
ΔP	11,475	7,475	3,475	-0,525	-0,525	-0,525	-0,525
$\Delta V=\Delta S+h\Delta P$	40,540	26,890	13,240	-0,410	3,590	7,590	11,590

(d)

	Rate	t=0	t=1	Capital transactions
Take a loan, million USD	5%	9,524	10,000	issue securities
Make a deposit, million PLN	12%	38,095	42,667	buy domestic T-bill
FX rate (PLN/USD		4,000	4,267	

1.2 Performance Attribution

Measuring performance attribution for actively managed portfolios is theoretically simple. The total value added from active management can be calculated by comparing the portfolio's return with the portfolio's benchmark return. Decomposition of the value added into components to determine sources of increase or decline in value is more difficult. The sources of value added may decomposed to

- active asset management (selection of foreign assets),
- asset allocation (structure of foreign assets),
- currency allocation (selection of foreign currencies).

Two techniques of the performance attribution used in practice include

- absolute contribution analysis,
- relative contribution analysis.

Both techniques identify the contribution of the above stated sources of value added to the performance of a currency portfolio and both compare the realized return with the benchmark portfolio. If the currency portfolio in time t=1 is compared to the currency portfolio in time t=0 the decomposition of value added does not show in detail the whole history of contribution. Both techniques give the same results, but they partition the same value added in different way.

The realized total currency portfolio return is

(16) $$r_P = \sum_i w_i \left[r_z^i + f_i \right] + \sum_j H_j \left[r_d^j - f_j \right]$$

where

w_i - the portfolio weight held in asset i,

r_z^i - the asset return of asset i,

f_i - the forward premium for currency i,

H_j - the currency exposure of the portfolio to currency j,

r_d^j - the currency return for currency j.

The return on the benchmark portfolio (all elements are with $\sim$ sign) can be calculated as

(17) $$\tilde{r}_P = \sum_i \tilde{w}_i \left[\tilde{r}_z^i + f_i \right] + \sum_j \tilde{H}_j \left[r_d^j - f_j \right] = \tilde{r}_P^z + \tilde{r}_P^d$$

where

(18) $$\tilde{r}_P^z = \sum_i \tilde{w}_i \left[\tilde{r}_z^i + f_i \right]$$ - the required (benchmark) hedged asset return,

(19) $$\tilde{r}_P^d = \sum_j \tilde{H}_j \left[r_d^j - f_j \right]$$ - the benchmark return for the currency j.

Let the difference between the realized asset return and the benchmark asset return be

(20) $$\Delta r_z^i = r_z^i - \tilde{r}_z^i$$

We can also write the changes in portfolio weights and currency exposure as

(21) $$\Delta w_i = w_i - \tilde{w}_i$$

(22) $$\Delta H_j = H_j - \tilde{H}_j$$

Absolute contribution analysis decomposes the value added (more exactly the difference between the total portfolio return and the total benchmark portfolio return) in the following way

$$(23) \quad r_P - \tilde{r}_P = \sum_i w_i \Delta r_z^i + \sum_i \Delta w_i \left[\tilde{r}_z^i + f_i\right] + \sum_j \Delta H_j \left[r_d^j - f_j\right] + \varepsilon$$

$\sum_i w_i \Delta r_z^i$ - active asset management,

$\sum_i \Delta w_i \left[\tilde{r}_z^i + f_i\right]$ - active asset allocations,

$\sum_j \Delta H_j \left[r_d^j - f_j\right]$ - active currency allocation.

The last error term ε captures the cross-products between asset returns and currency returns and is relatively small.

Active asset management gives positive value added when selected assets including financial instruments have higher returns than the assets in the benchmark portfolio. It is reasonable to select assets with greater returns but it is also important to keep in mind that higher return of asset is accompanied by greater credit risk. The conservative policy (selection of credit risk-free government securities) will produce tiny results.

Active asset allocation evaluates the changes in the structure of portfolio. The best results are obtained when the weights of the highest hedged returns are increased.

Active currency allocation involves selection of currencies and the appropriate currency exposures which is obtained using the hedge ratios and derivatives. The value added depends on hedge ratios and also the forward contract return. The last component depends on currency returns which are not dependent on a company wishes or decisions. But a conservative company using regularly hedged currency positions ($H=0$, $\Delta H=0$) may almost completely eliminate profits and losses from the currency allocation.

Relative contribution analysis represents an alternative way of measuring the same value added. The active asset management component is not changed. But this technique changes the way the individual value added by active asset allocation and active currency allocation. These values are calculated using the aggregate benchmark return for assets and the aggregate benchmark return for currencies. Such rearrangement changes the individual returns. The value added is

$$(24) \quad r_P - \tilde{r}_P = \sum_i w_i \Delta r_z^i + \sum_i \Delta w_i \left[\tilde{r}_z^i + f_i - \tilde{r}_P^z\right] + \sum_j \Delta H_j \left[r_d^j - f_j - \tilde{r}_P^d\right] + e$$

Problem 2. The currency performance attribution

Consider the performance of a simple portfolio composed of domestic bonds (position 1) and three investments in three different currencies (posiitions 2, 3, and 4).

Realized Strategy

Position	1	2	3	4		
The local asset return of asset i						
r_z^i	2,00%		1,00%	5,00%		
The currency return for currency j						
r_d^j		-1,00%	2,00%	3,00%		
The forward premium for currency i						
f_i		0,10%	0,10%	0,10%		
The currency exposure of the portfolio to currency j						
H_j		-0,50	0,10	0,50		
The portfolio weights held in asset i						
w_i	40%	10%	20%	30%		
The realized return for the hedged assets						
$w_i\left[r_z^i + f_i\right]$	0,80%	0,01%	0,22%	1,53%	$\sum_i w_i\left[r_z^i + f_i\right]$	2,56%
The ralized return for the currency						
$H_j\left[r_d^j - f_j\right]$		0,55%	0,19%	1,45%	$\sum_j H_j\left[r_d^j - f_j\right]$	2,19%
Total return	0,80%	0,56%	0,41%	2,98%	$r_P =$	4,75%

The Benchmark Stategy

The local asset return of asset i						
$\tilde{r}_z^i$	1,00%	1,00%	1,00%	1,00%		
The currency exposure of the portfolio to						
$\tilde{H}_j$		-0,10	0,10	0,10		
The portfolio weights						
$\tilde{w}_i$	25,0%	25%	25,0%	25,0%		
The benchmark return for the assets						
$\tilde{w}_i\left[\tilde{r}_z^i + f_i\right]$	0,25%	0,28%	0,28%	0,28%	$\tilde{r}_P^z = \sum_i \tilde{w}_i\left[\tilde{r}_z^i + f_i\right]$	1,08%
The benchmark return for the currency						
$\tilde{H}_j\left[r_d^j - f_j\right]$		0,11%	0,19%	0,29%	$\tilde{r}_P^d = \sum_j \tilde{H}_j\left[r_d^j - f_j\right]$	0,59%
Total return	0,25%	0,39%	0,47%	0,57%	$\tilde{r}_P =$	1,67%
Total value added					$r_P - \tilde{r}_P =$	3,09%

(a) Show the absulute contribution analysis results.
(b) Show the relative contribution analysis results.

Solution
(a)
Absulute Contribution Analysis

Active asset management

$w_i \Delta r_z^i$	0,40%	-0,10%		1,20%	$\sum_i w_i \Delta r_z^i$	1,50%

Active asset allocation

$\Delta w_i\left[\tilde{r}_z^i + f_i\right]$	0,15%	-0,17%	-0,06%	0,06%	$\sum_i \Delta w_i\left[\tilde{r}_z^i + f_i\right]$	-0,02%

Active currency allocation

$\Delta H_j\left[r_d^j - f_j\right]$		0,44%		1,16%	$\sum_j \Delta H_j\left[r_d^j - f_j\right]$	1,60%

Total value added	0,55%	0,18%	-0,06%	2,42%	$r_P - \tilde{r}_P =$	3,09%

(b)
Relative Contribution Analysis

Active asset management

$w_i \Delta r_z^i$	0,40%	-0,10%		1,20%	$\sum_i w_i \Delta r_z^i$	1,50%

Active asset allocation

$\Delta w_i\left[\tilde{r}_z^i + f_i - \tilde{r}_P^z\right]$	-0,01%	0,00%	0,00%	0,00%	$\sum_i \Delta w_i\left[\tilde{r}_z^i + f_i - \tilde{r}_P^z\right]$	-0,02%

Active currency allocation

$\Delta H_j\left[r_d^j - f_j - \tilde{r}_P^d\right]$		0,68%		0,92%	$\sum_j \Delta H_j\left[r_d^j - f_j - \tilde{r}_P^d\right]$	1,60%

Total value added	0,39%	0,57%	0,00%	2,13%	$r_P - \tilde{r}_P =$	3,09%

1.3 Theoretical Hedge Ratios

1.3.1 Forward and Futures

Theoretical hedge ratios apply the relationship between the forward price and the spot price. Assume that the hedged position is formed by holding the underlying asset and selling h futures contracts

(25) $V_t = S_t - h\,(F - F_t)$
and
(26) $V_{t+1} = S_{t+1} - h\,(F - F_{t+1})$
where:
S_t – the spot price,
F_t – the futures price,
h – hedge ratio (negative for the short).

The change in the value of the hedged position is:

(27) $\Delta V = \Delta S + h\Delta F$

The hedge ratio would be

(28) $$h = \frac{\Delta V - \Delta S}{\Delta F}$$

For a theoretical, or delta nautral hedge $\Delta V = 0$ and the hedge ratio is

(29) $$h = \frac{-\Delta S}{\Delta F}$$

1.3.2 Currency Forward and futures

The change in the futures contract's price relative to the spot exchange rate is

(30) $$\Delta F = \Delta S_0 \frac{\left(1 + i_d^N T\right)}{\left(1 + i_f^N T\right)}$$

The hedge ratio can be calculated as:

(31) $$h = -\frac{\Delta S_0}{\Delta F} = -\frac{\left(1 + i_f^N T\right)}{\left(1 + i_d^N T\right)}$$

Problem 3. The equal-dollar-matched hedge ratio

An investor will receive a payment of $1 million from a major export contract in 30 days. The current spot exchange rate is 4,5709 PLN/$, but investor is concerned that the dollar will depreciate. The dollar futures contract is priced now at 4,6400 PLN/$.

(a) How many contracts would need to be used to hedge the decline in value assuming that the contract size is $10000.

(b) If the spot exchange rate subsequently declines to 3,8000 PLN/$ and the futures price declines to 3,8603 PLN/$, what is the net result of the hedge for the investor.

Solution

(a)

If the zloty value of the dollar falls, the anticipated payment in zloties will be worth less in zloties. To hedge against currency risk, the investor would need to sell 100 futures contracts. The hedge ratio is h = -1.

$$n = \frac{hK}{k} \qquad = \qquad 100 \text{ contracts}$$

(b)

	Exchange rate	Value $ millions	PLN millions
Spot exchange rate t=0	4,5709	1,000	4,5709
Furtures exchange rate t=0	4,6400	1,000	4,6400
Spot exchange rate t=1	3,8000	1,000	3,8000
Futures exchange rate t=1	3,8603	1,000	-3,8603
Value of the hedged position			4,5797

The value of the hedged position at time t is equal to the initial futures price plus the basis at time t:

$(F + (S_t - F_t)) * K =$ (4,6400 + 3,8000- 3,8603) * 1 = 4,5797 PLN million.

The value of the hedged position would be the same as the original forward position if the basis had closed to zero at the termination of the futures contract.

Problem 4. Theoretical hedge ratio. Currency hedge

An investor wants to hedge a \$1 million currency gap against currency risk.
The currency futures expire in 78 days. The current home risk-free interest rate is 18,0%, and foreign risk-free interest rates is 6,0%. The contract size is \$ 10 000.
(a) Calculate a delta neutral hedge ratio.
(b) Calculate the number of contracts required to hedge the position.

Solution

(a)

The hedge ratio for the equity porfolio is $h = -\frac{\Delta S_0}{\Delta F} = -\frac{\left(1 + i_f^N T\right)}{\left(1 + i_d^N T\right)}$ = -0,975

(b)

$$n = \frac{hK}{k} \quad = \quad -97,5$$

The currency gap should be hedged by selling 97 currency futures contracts.