1. Currency Exposure. VaR for currency positions. Hedged and unhedged positions

Currency Exposure

Currency exposure represents the relationship between stated financial goals and exchange rate movements, instruments and currency selection and portfolio structure.

Currency Risk Measures

Statistical Measures

A number of different statistics can be applied to measure currency risk. These include variances, standard deviations, percentiles, coefficients of variation.

Currency log return distribution is very close to symmetric normal distribution. Thus variance or standard deviation are good currency risk measures. Variance for the currency portfolio is computed using the correlation between returns on exchange rate (appreciation and depreciation rates) and returns on foreign assets. The variance of currency returns is also used to compute risk diversification effects.

Var, EaR, CFaR – variance and covariance method

In this approach log currency returns (continuously compounded returns) are used in temporal and cross-section aggregation. It is assumed that log returns are normally distributed. In a portfolio all returns are multivariate normally distributed. The sum of normal random variables is itself normally distributed. The entire distribution of returns can be characterized by two parameters: mean and variance. These parameters are usually forecasted using historical data.

Such assumptions are not suitable for portfolios that contain options.

Financial return distributions often have fat tails. Extreme returns movements occur more frequently than implied by normal distribution and the peak of the return distribution is higher and narrower (leptokurtotic distribution).

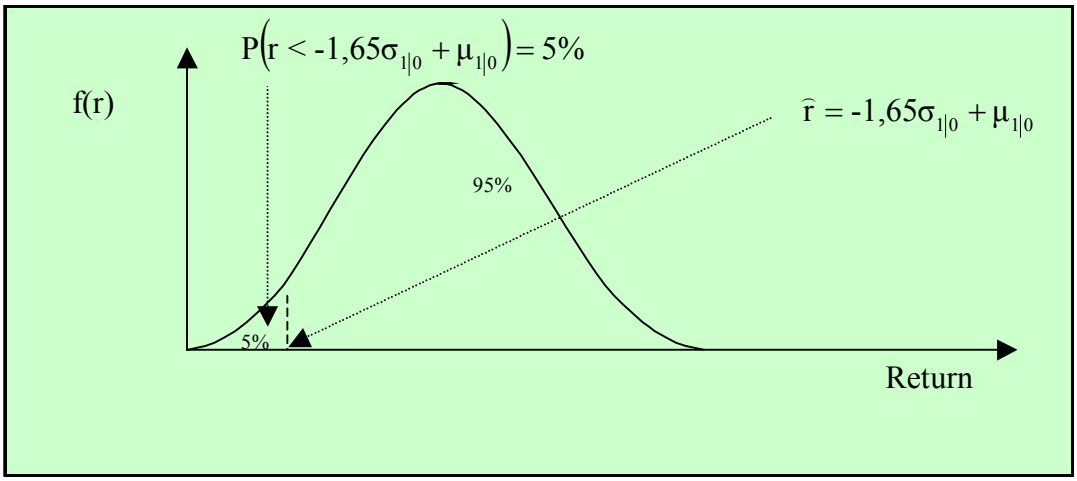

Figure 1. Return Distribution

We want to find the 5th percentile of return. There is a 5 percent probability that an observed return at time t is less than –1.65 times its standard deviation plus its mean. Assuming that mean=0 (one day horizon) we have

(1) $$P\{r_t \le -1{,}65\sigma_{t|t-1}\} = 5\%$$

where
-1,65 – 5th percentile of the standard normal distribution
$\sigma_{t|t-1}$ – standard deviation of continuously compounded returns for time t prepared at time t-1.

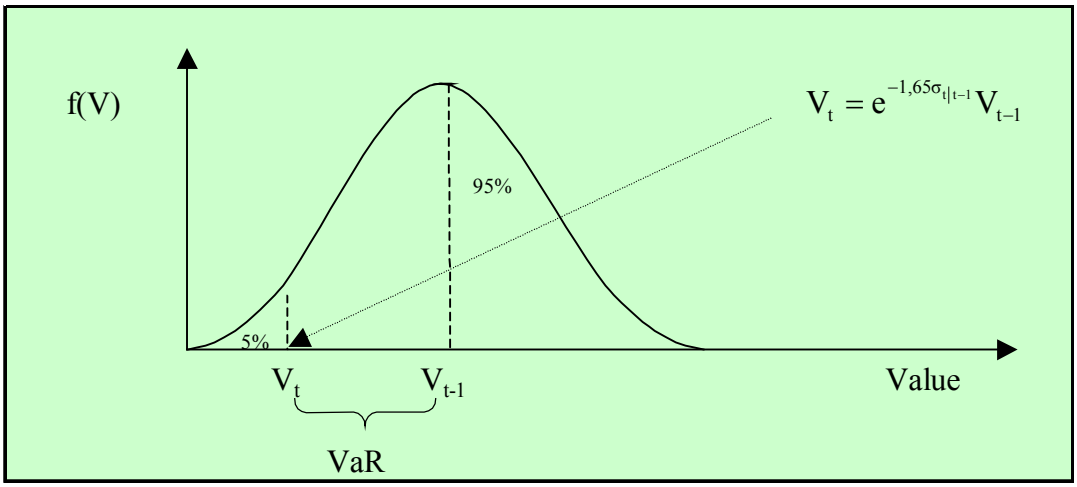

Figure 2. Value at Risk

VaR of a single asset can be calculated as follows

(2) $$VaR = \left[1 - e^{-1{,}65\sigma_{t|t-1}}\right] V_{t-1}$$

where
e – Euler's constant (e=2,7183),
V_{t-1} – portfolio value at time t-1.

or, using the common approximation

(3) $$VaR \cong 1{,}65\sigma_{t|t-1} V_{t-1}$$

Problem 1. VaR – one asset

Suppose you hold a $25 million position and current spot exchange rate is 4 PLN/USD.
Expected return over a 1-day horizon period is equal to 0.
Expected volatility is equal to 1%.
(a) What is the current market value of this position. What is the forecasted value ?
(b) Make a forecast of the 1-day return on the portfolio, such that there is 5% chance that the realized return will be less than forecasted return.
(c) Calulate the portfolio's "worst case" value based on the forecasted rate (b).
(d) Calculate VAR.
(e) Calculate VaR using simple approximation.

Solution

(a)

The current mark-to-market value is 100 million zloties.

The forecasted future value V_1 is $V_1 = V_0 e^r$

(b)

We assume that the ralized return will be less than the forecasted return with a 5% probability.

$$P\left(r < \hat{r}\right) = 5\%$$

$$P\left(r < -1{,}65\sigma_{1|0} + \mu_{1|0}\right) = 5\%$$

We assume that the expected return over a 1 day horizon is equal to 0.

$$\mu_{1|0} = 0$$

The forecasted return is

$$\hat{r} = -1{,}65\sigma_{1|0} \quad = \quad -1{,}645\%$$

(c)

The portfolio "worst case" value is

$$\hat{V}_1 = V_0 e^{\hat{r}} \quad = \quad 98{,}369 \quad \text{million zloties}$$

(d)

The Value at Risk is

$$VaR = V_0 - \hat{V}_1 = V_0\left(1 - e^{\hat{r}}\right) \quad = \quad 1{,}631 \quad \text{million zloties}$$

(e)

VaR is approximately equal to

$\alpha/2$	t	$t\,\sigma$	V_0	$VaR = t\,\sigma\,V_0$
5,00%	1,645	0,01645	100	1,645 million zloties

Problem 2. VaR – two assets

Suppose you hold a long currency position $ 25 million in the 10-year US bond and the current spot exchange rate is 4,00 PLN/USD.
Currency return is a random variable which follows conditional normal distribution with the following parameters: mean = 0; standard deviation 1,0%.
(a) What is the current value of this position in zloties ?
(b) What is your VaR over a 1-day horizon period, given there is a 5%, 2,5%, or 0,5% chance of understating the realized loss.
(c) Calculate VAR, when you estimate the expected volatility of a bond equal to be 0,5%, and the correlation between the return on the PLN/USD exchange rate and the 10-year US bond to be -0,5.

Solution

(a)

The currency exposure is 100 million PLN.

(b)

VaR is appriximately equal to $\text{VaR} \cong t_{\frac{\alpha}{2}} \sigma_{t|t-1} V_{t-1}$

$\alpha/2$	$t_{\frac{\alpha}{2}}$	$\sigma_{t\|t-1}$	V_{t-1}	VaR	
5,00%	1,645	1,0%	100	1,645	million PLN
2,50%	1,960	1,0%	100	1,960	million PLN
0,50%	2,576	1,0%	100	2,576	million PLN

(c)

We have the following information

1. standard deviation of the currency return $\sigma_{r_d} = 1,0\%$
2. 10-year US bond standa deviation $\sigma_{r_z} = 0,5\%$
3. correlation coefficient $\rho_{r_z r_d} = -0,5$

The variance of returns is equal to

$$\sigma^2_{r_H} = \sigma^2_{r_z} + \sigma^2_{r_d} + 2\rho_{r_z r_d}\sigma_{r_z}\sigma_{r_d} = 0,008\%$$

The standard deviation is $\sigma_{r_H} = 0,866\%$

We compute VaR using formula $\text{VaR} \cong t_{\frac{\alpha}{2}} \sigma_{r_H} V_{t-1}$

$\alpha/2$	$t_{\frac{\alpha}{2}}$	σ_{rH}	V_{t-1}	VaR	
5,00%	1,645	0,866%	100	1,424	million PLN
2,50%	1,960	0,866%	100	1,697	million PLN
0,50%	2,576	0,866%	100	2,231	million PLN

Currency Gap

Statistical measures including Var are not sufficient currency risk measures. It is also very important to calculate currency gap. All assets and liabilities in a given currency and also all positions in derivatives (currency forward, futures, options and swaps) must be included in calculating currency gaps.

Long and short position

An investor has a long position in a currency when he holds more assets than liabilities in this currency. A short position exists when foreign liabilities exceed foreign assets.

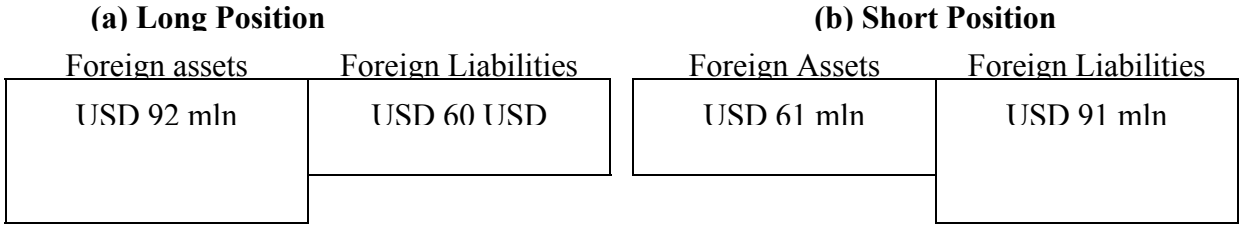

Figure 3. Long and Short Currency Position

In Figure 3 a, net long position is equal to $32 million. An investor faces the risk that the foreign currency will fall in value against zloty (dollar depreciates, zloty appreciates). The value of such position decreases in zloties. When dollar appreciates (zloty depreciates) net long position in a currency is worth more in zloties.

In Figure 3 b, net short position is equal to -$30 million. An investor faces the risk that the foreign currency will rise in value against zloty (dollar appreciates, zloty depreciates). The value of such position decreases in zloties. When dollar depreciates (zloty appreciates) net short position in a currency is worth more in zloties.

Table 1. Profits and Losses on Foreign Currency Positions

Foreign currency	Long Position	Short Position
Appreciation	Profit	Loss
Depreciation	Loss	Profit

Currency gap in a given currency

Currency gap in a given currency j measured in domestic currency (zloties) is

(4) $$L_j = \left[AN_j + ZN_j\right]S_{t-1,j}$$

AN_j – net assets (assets - liabilities) in currency j,
ZN_j – net FX bought (FX bought – FX sold),
$S_{t-1,j}$ – spot exchange rate.

Total currency gap

Total currency gap is calculated as follows:

(5) $$L_w = \sum_{j=1}^{n} \left[AN_j + ZN_j\right]S_{t-1,j}$$

Gains or losses on foreign currency positions depend on currency gaps ad foreign exchange rate movements (currency returns). Change in value of position in currency j is

(6) $$\Delta W_j = L_j r_{dj}$$

where:
r_{dj} – currency return (rate of appreciation, rate of depreciation),

Total change in value for the overall currency portfolio depends on all currency gaps and all currency returns.

(7) $$\Delta W = \sum_{j=1}^{n} \Delta W_j = \sum_{j=1}^{n} L_j r_{dj}$$

Problem 3. Currency Gap

The foreign assets are 10 million USD, foreign liabilites 11 million USD.
Net FX bought 3 million USD. The foreign exchange rate is 4,00 PLN/USD.
(a) Calculate the net currency position.
(b) What is the FX loss/gain as a result of the exchange rate shocks +-10%.

Solution

(a)

Net position

2 000 000 USD
8 000 000 PLN

(b)

FX	Return	ΔW	W
3,6	-10%	-800 000	7 200 000
4,0	0%	0	8 000 000
4,4	10%	800 000	8 800 000

Problem 4. Spot Position. Forward position

Today's spot exchange rate is 4,00 zł/USD. The treasurer of a company expects to receive a payment $100000 from a foreign customer in the next 60 days. Two-month forward contracts are quoted at a rate 4,05 PLN/USD.
(a) Describe the current currency position.
(b) Indicate whether a company should use a long or short forward contract to hedge against currency risk.
(c) Discuss the unhedged position, forward position and hedged position at expiration
(i) if the spot rate is 4,30 PLN/USD.
(ii) if the spot rate is 3,90 PLN/USD.

Solution

(a)
A company takes a long currency position.

(b)
The company has to go short a forward contract, or sell $100000 at a price of 4,05 PLN/USD. A two-month hedge would be constructed by selling dollars with two months to expiration. Selling currency forward eliminates the currency risk.

(c) Expiration date
(I) The spot exchange rate is 4,30 PLN/USD

Unhedged position
The company receives the payment from a foreign customer

100000	x	4,30	=	430000 PLN

Positive exchange rate adjustments are equal to 30000 PLN (extra profits before tax).

Forward contract
The company is obliged to sell $100000 to the dealer and is paid 4,05 PLN per USD for a total payment

100000	x	4,05	=	405000 PLN

If non-delivery forward contract was agreed, the dealer receives the difference between the spot and the forward rate exchange rate. The transaction is settled in cash.

100000	x	0,25	=	25000 PLN

The net effect for the company from the forward transaction (loss with a minus sign) is:
K = (forward rate - spot rate) x amount
As the forward rate is lower than the spot rate, the company suffers a loss
K = (4,05 PLN/USD - 4,30 PLN/USD) x 100000 USD = -25000 PLN.

Hedged position
A typical hedging situation is when long (or short) curence position is supplemented with a short (or long) forward position. The company locks in the exchange rate at which it will convert the dollars. Regardless of the exchange rate at expiration the company has a gain equal to 5000 PLN.

(ii) The spot exchange rate is 3,90 PLN/USD

Unhedged position

The company receives the payment from a foreign customer

100000 x 3,90 = 390000 zł

Negative exchange rate adjustments are equal to -10000 (losses before tax).

Forward contract

The company is obliged to sell $100000 to the dealer and is paid 4,05 PLN per USD for a total payment

100000 x 4,05 = 405000 zł

If non-delivery forward contract was agreed, the dealer pays the difference between the spot and the forward rate exchange rate. The transaction is settled in cash.

100000 x 0,15 = 15000 zł

The net effect for the company from the forward transaction (loss with a minus sign) is:

K = (forward rate - spot rate) x amount

As the forward rate is larger than the spot rate, the company has a gain

K = (4,05 PLN/USD - 3,90 PLN/USD) x 100000 USD = 15000 PLN.

Hedged position

The company locks in the exchange rate at which it will convert the dollars.

Regardless of the exchange rate at expiration the company has a gain equal to 5000 PLN.