1. Credit Exposure. Simulation Methods

Credit risk exposure may be analysed with DCF methods (traditional and arbitrage risk-free, contingent claim methods and simulation methods.

Credit exposure

Adjusted cash flows

$$PV_j = \frac{p_w CF_1}{(1+RRR_p)^1} + \frac{p_w^2 CF_2}{(1+RRR_p)^2} + \ldots + \frac{p_w^n CF_n}{(1+RRR_p)^n}$$

R_p - risk-free rate

p_w - probability of repayment

Adjusted interest rates

$$PV_j = \frac{CF_1}{(1+RRR_p+q)^1} + \frac{CF_2}{(1+RRR_p+q)^2} + \ldots + \frac{CF_n}{(1+RRR_p+q)^n}$$

R_k - interest rate with credit risk premium = R_p + q

q - credit risk premium

Figure 1. Credit Risk Exposure

1.1.1 Traditional Valuation – with adjusted cash flows

Relation between credit risk premium and probability of repayment

Credit quality changes may influence interest rate (credit downgrades increase credit premiums) or cash flows (credit downgrades decrease expected cash flows). Credit quality changes cannot be reflected in both interest rate adjustment and cash flow adjustment.

The interest rate RRR_j which includes credit risk premium depends on the credit risk-free rate RRR_p as follows:

(1) $$RRR_j = \frac{1+RRR_p}{p_w} - 1$$

Credit risk premium is equal to:

(2) $$q = RRR_j - RRR_p = \frac{(1-p_w)(1+RRR_p)}{p_w}$$

Probability of repayment for a given credit risk premium is:

(3) $$p_w = \frac{1+RRR_p}{1+RRR_p+q} = \frac{1+RRR_p}{1+RRR_j}$$

Problem 1. Valuation with credit risk

A loan amount is 1000 PLN. Initial provision is 1%. Maturity is 5 years. The agreed with a borrower fixed interest rate is 18,00%. The risk-free interest rate is 16,11%. Probability of repayment of the borrower after one year is 98%. The loan is serviced with a traditional method (equal repayments, interests calculated on outstanding debt).

(a) Calculate the required rate of return for a bank (cost of debt for a borrower) which includes credit risk premium.

(b) Calculate the loan value using the risk free rate.

(c) Calculate the loan value using interest rate that includes credit risk premium.

Solution

(a)

Interest rate including risk premium

$$RRR_j = \frac{1 + RRR_p}{p_w} - 1 \qquad = \qquad 18{,}48\%$$

Risk premium

$$q = RRR_j - RRR_p = \frac{(1 - p_w)(1 + RRR_p)}{p_w} \qquad = \qquad 2{,}37\%$$

(b)

	0	1	2	3	4	5
Interests		180,00	144,00	108,00	72,00	36,00
Repayment		200,00	200,00	200,00	200,00	200,00
Cash flows		380,00	344,00	308,00	272,00	236,00
Probability of repayment		98,0%	96,0%	94,1%	92,2%	90,4%
Adjusted cash flows		372,40	330,38	289,89	250,88	213,33
Discounting factor at 16,11%		0,8612	0,7417	0,6388	0,5501	0,4738
Discounted cash flow		320,72	245,04	185,17	138,01	101,07
Cumulative cash flow		320,72	565,75	750,92	888,93	990,00

Loan value is 990 PLN.

(c)

	0	1	2	3	4	5
Cash flows		380,00	344,00	308,00	272,00	236,00
Discounting factor at 18,48%		0,8440	0,7123	0,6012	0,5074	0,4282
Discounted cash flow		320,72	245,04	185,17	138,01	101,07
Cumulative cash flow		320,72	565,75	750,92	888,93	990,00

Loan value is 990 PLN.

1.1.2 Non-arbitrage valuation with spot rates

Problem 2. Valuation with Spot Rates

The five possible loans have the same amount 1000 PLN. Each loan is serviced with a traditional method. The fixed interest rates and initial provisions for different maturities are as follows:

Maturity	1	2	3	4	5
Fixed interest rate	20,0%	19,5%	19,0%	18,5%	18,0%
Provision	1,0%	1,0%	1,0%	1,0%	1,0%

(a) Calculate and interpret internal rates of return, spot rates and loan values using IRR and spot rates for each loan.
(b) Calculate forward rates. Draw the term structure of fixed interest rates, IRR, spot rates and forward rates.
(c) For a loan with a 5 years maturity calculate interests and loan value using IRR and spot rates.

(a)

	0	1
Debt	1000,00	0,00
Interests		200,00
Repayments		1000,00
Cash flows	-990,00	1200,00
IRR		21,21%
Discounting factor		0,8250
Discounted cash flow		990,00
Cumulative cash flow		990,00
Spot rate		21,21%
Discounting factor		0,8250
Discounted cash flow	-990,00	990,00
Cumulative cash flow		990,00

	0	1	2
Debt	1000,00	500,00	0,00
Interests		195,00	97,50
Repayments		500,00	500,00
Cash flows	-990,00	695,00	597,50
IRR		20,35%	20,35%
Discounting factor		0,8309	0,6904
Discounted cash flow		577,48	412,52
Cumulative cash flow		577,48	990,00
Spot rate		21,21%	19,76%
Discounting factor		0,8250	0,6973
Discounted cash flow	-990,00	573,38	416,63
Cumulative cash flow		573,38	990,00

	0	1	2	3
Debt	1000,00	666,67	333,33	0,00
Interests		190,00	126,67	63,33
Repayments		333,33	333,33	333,33
Cash flows	-990,00	523,33	460,00	396,67
IRR		19,67%	19,67%	19,67%
Discounting factor		0,8356	0,6983	0,5835
Discounted cash flow		437,32	321,22	231,46
Cumulative cash flow		437,32	758,54	990,00
Spot rate		21,21%	19,76%	18,65%
Discounting factor		0,8250	0,6973	0,5987
Discounted cash flow	-990,00	431,75	320,75	237,50
Cumulative cash flow		431,75	752,50	990,00

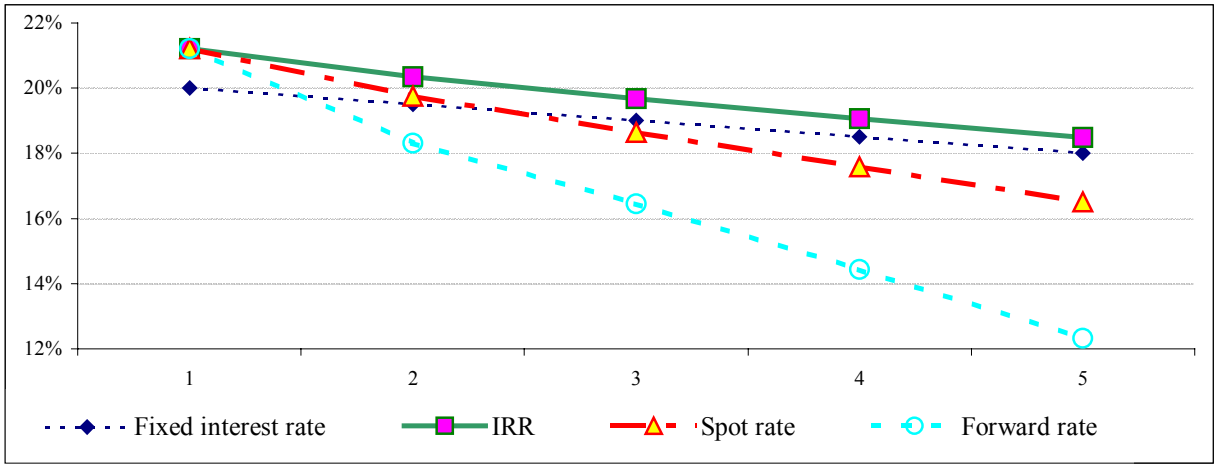
	0	1	2	3	4
Debt	1000,00	750,00	500,00	250,00	0,00
Interests		185,00	138,75	92,50	46,25
Repayments		250,00	250,00	250,00	250,00
Cash flows	-990,00	435,00	388,75	342,50	296,25
IRR		19,06%	19,06%	19,06%	19,06%
Discounting factor		0,8399	0,7055	0,5925	0,4977
Discounted cash flow		365,37	274,25	202,94	147,44
Cumulative cash flow		365,37	639,62	842,56	990,00
Spot rate		21,21%	19,76%	18,65%	17,58%
Discounting factor		0,8250	0,6973	0,5987	0,5232
Discounted cash flow	-990,00	358,88	271,07	205,07	154,99
Cumulative cash flow		358,88	629,94	835,01	990,00

	0	1	2	3	4	5
Debt	1000,00	800,00	600,00	400,00	200,00	0,00
Interests		180,00	144,00	108,00	72,00	36,00
Repayments		200,00	200,00	200,00	200,00	200,00
Cash flows	-990,00	380,00	344,00	308,00	272,00	236,00
IRR		18,48%	18,48%	18,48%	18,48%	18,48%
Discounting factor		0,8440	0,7123	0,6012	0,5074	0,4282
Discounted cash flow		320,72	245,04	185,17	138,01	101,07
Cumulative cash flow		320,72	565,75	750,92	888,93	990,00
Spot rate		21,21%	19,76%	18,65%	17,58%	16,51%
Discounting factor		0,8250	0,6973	0,5987	0,5232	0,4658
Discounted cash flow	-990,00	313,50	239,86	184,41	142,30	109,92
Cumulative cash flow		313,50	553,36	737,78	880,08	990,00

The loan value is 990 PLN.

(b)

	1	2	3	4	5
Fixed interest rate	20,00%	19,50%	19,00%	18,50%	18,00%
IRR	21,21%	20,35%	19,67%	19,06%	18,48%
Spot rate	21,21%	19,76%	18,65%	17,58%	16,51%
Forward rate	21,21%	18,32%	16,46%	14,45%	12,32%



(c)

	0	1	2	3	4	5
Debt	1000,00	800,00	600,00	400,00	200,00	0,00
Interests		212,12	146,53	98,75	57,78	24,65
Repayments		200,00	200,00	200,00	200,00	200,00
Prowizja	10,00					
Cash flows	-990,00	412,12	346,53	298,75	257,78	224,65
IRR		19,05%	19,05%	19,05%	19,05%	19,05%
Discounting factor		0,8400	0,7056	0,5927	0,4978	0,4182
Discounted cash flow		346,17	244,50	177,05	128,33	93,94
Cumulative cash flow		346,17	590,68	767,73	896,06	990,00
Spot rate		21,21%	19,76%	18,65%	17,58%	16,51%
Discounting factor		0,8250	0,6973	0,5987	0,5232	0,4213
Discounted cash flow	-990,00	340,00	241,63	178,87	134,86	94,63
Cumulative cash flow		340,00	581,63	760,50	895,37	990,00

The loan value is 990 PLN.

Valuation with forward rates

Forward rates

(4) $$ {}_{T+k}f_T = \left[\frac{(1+z_{T+k})^{T+k}}{(1+z_T)^T} \right]^{\frac{1}{k}} - 1 $$

Problem 3. Spot Position. Forward position

A loan amount is 1000 PLN. Initial provision is 1%. Maturity is 5 years. The agreed with the borrower fixed interest rate is 18,00%. The loan is serviced with a traditional method. The spot rates are as follows:

1	2	3	4	5
21,21%	19,76%	18,65%	17,58%	16,51%

(a) Calculate the loan value at time t=0 using spot rates.
(b) Calculate the loan value at time t=1 using forward rates.

Solution

(a)

	0	1	2	3	4	5
Interests		180,00	144,00	108,00	72,00	36,00
Repayment		200,00	200,00	200,00	200,00	200,00
Cash flows		380,00	344,00	308,00	272,00	236,00
Spot rate		21,21%	19,76%	18,65%	17,58%	16,51%
Discounting rate		0,8250	0,6973	0,5987	0,5232	0,4658
Discounted cash flow		313,50	239,86	184,41	142,30	109,92
Cumulative cash flow		313,50	553,36	737,78	880,08	990,00

The loan value is 990 PLN.

(b)

	0	1	2	3	4	5
Cash flows		380,00	344,00	308,00	272,00	236,00
Forward rate			18,32%	17,38%	16,40%	15,36%
Discounting rate		1,0000	0,8452	0,7257	0,6341	0,5646
Discounted cash flow		380,00	290,74	223,53	172,49	133,24
Cumulative cash flow		380,00	670,74	894,28	1066,76	1200,00

The loan value at t=1 (a moment before interests and the first repayment at time t=1) is 1200 PLN. It is obvious that the same value is received by compounding the present value of a loan using a one-year spot rate, that is 990 * (1+21,21%) = 1200 PLN.

1.1.3 Symulation

Problem 4. Simulation of credit risk

A portfolio of loans is grouped according to credit ratings of the borrowers. Maturities, fixed interest rates, recovery rates and volatilities of recovery rates are as follows:

	Amount	Maturity	Interest rate	Recovery rates	Volatility
AAA	20,000	4	12,00%	0,7	0,35
AA	40,000	2	12,00%	0,7	0,35
A	100,000	3	14,00%	0,7	0,35
BBB	200,000	5	15,00%	0,6	0,3
BB	80,000	4	15,00%	0,6	0,3
B	60,000	3	16,00%	0,6	0,3
CCC	20,000	2	20,00%	0,5	0,25
D	0,000	2	30,00%	0,5	0,25
Total	520,000				

The migration matrix is

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	90,82%	8,26%	0,74%	0,06%	0,11%	0,00%	0,00%	0,00%
AA	0,65%	90,88%	7,69%	0,58%	0,05%	0,13%	0,02%	0,00%
A	0,08%	2,42%	91,30%	5,23%	0,68%	0,23%	0,01%	0,05%
BBB	0,03%	0,31%	5,87%	87,46%	4,96%	1,08%	0,12%	0,17%
BB	0,02%	0,12%	0,64%	7,71%	81,16%	8,40%	0,98%	0,98%
B	0,00%	0,10%	0,24%	0,45%	6,86%	83,50%	3,92%	4,92%
CCC	0,21%	0,00%	0,41%	1,24%	2,67%	11,70%	64,48%	19,29%
D	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Spot interest rates are

Year	AAA	AA	A	BBB	BB	B	CCC	D
1	12,00%	12,15%	12,30%	12,45%	12,60%	12,75%	21,38%	
2	12,50%	12,65%	12,80%	12,95%	13,10%	13,25%	20,63%	
3	13,00%	13,15%	13,30%	13,45%	13,60%	13,75%	19,88%	
4	13,50%	13,65%	13,80%	13,95%	14,10%	14,25%	19,13%	
5	14,00%	14,15%	14,30%	14,45%	14,60%	14,75%	18,38%	

(a) Calculate the values of loans at time t=1.

(b) Calculate the mean values, variances, standard deviations and percentiles (1%).

(d) Calculate the asset return tresholds for the migration matrix.

(d) Show the simulation results (correlation coefficient =0,2).

Solution

(a)

Values at t=1

Value	AAA	AA	A	BBB	BB	B	CCC	D
AAA	21,963	21,894	21,825	21,756	21,687	21,619	19,030	14,000
AA	44,800	44,747	44,693	44,640	44,587	44,534	41,710	28,000
A	116,574	116,318	116,062	115,808	115,554	115,302	103,883	70,000
BBB	239,875	238,963	238,057	237,156	236,261	235,372	206,963	120,000
BB	95,956	95,664	95,372	95,082	94,794	94,507	83,541	48,000
B	73,164	73,006	72,849	72,692	72,536	72,381	65,343	36,000
CCC	25,429	25,400	25,371	25,343	25,314	25,286	23,773	10,000
D	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

(b)

Results	Mean	Variance	St.dev.	Perc. 1%
AAA	21,96	0,001	0,02	21,89
AA	44,74	0,002	0,05	44,69
A	116,03	1,696	1,30	115,81
BBB	236,92	30,594	5,53	235,37
BB	94,23	28,039	5,30	83,54
B	70,33	78,781	8,88	36,00
CCC	21,36	35,990	6,00	10,00
D	0,00	0,000	0,00	0,00
Ogółem	605,56	498,403	22,32	

(c)

Asset return tresholds

	AA	A	BBB	BB	B	CCC	D
AAA	-1,33	-2,36	-2,92	-3,05	-5000000	-5000000	-5000000
AA	2,48	-1,37	-2,42	-2,87	-2,96	-3,53	-5000000
A	3,14	1,96	-1,54	-2,34	-2,76	-3,24	-3,29
BBB	3,42	2,71	1,54	-1,53	-2,21	-2,76	-2,93
BB	3,52	2,98	2,42	1,37	-1,26	-2,06	-2,33
B	4,27	3,08	2,70	2,41	1,43	-1,35	-1,65
CCC	2,87	2,87	2,50	2,08	1,69	0,99	-0,87
D	4,27	4,27	4,27	4,27	4,27	4,27	4,27

(d)
The distribution of the value pf credit portfolio i generated using simulation (10,000 iterations)
VaR: 605,4-574,5 = 30,9 PLN million.

	AAA	AA	A	BBB	BB	B	CCC	Total
Mean	22,0	44,7	116,0	236,9	94,2	70,2	21,4	605,4
Mode	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,9
Variance	0,0	0,0	1,1	31,0	25,3	67,1	30,7	176,5
Standard deviation	0,0	0,1	1,0	5,6	5,0	8,2	5,5	13,3
Skewness	-4,4	-47,8	-44,2	-20,5	-8,7	-3,8	-1,6	-4,2
Curtosis	30,6	2581,7	1967,5	428,5	79,4	16,1	3,5	34,0
Minimum Value	21,7	41,7	70,0	120,0	48,0	36,0	10,0	396,2
5% percentile	21,9	44,7	115,8	236,3	94,5	36,0	10,0	574,5
10% percentile	22,0	44,7	116,1	237,2	94,5	72,4	10,0	596,8
15% percentile	22,0	44,7	116,1	237,2	94,8	72,4	10,0	597,1
20% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	597,1
25% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	603,8
30% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,2
35% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,6
40% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,8
45% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,9
50% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,9
55% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,9
60% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,9
65% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,9
70% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	610,9
75% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	611,0
80% percentile	22,0	44,7	116,1	237,2	94,8	72,4	23,8	611,4
85% percentile	22,0	44,7	116,1	237,2	94,8	72,4	25,3	612,3
90% percentile	22,0	44,7	116,1	237,2	94,8	72,4	25,3	612,4
95% percentile	22,0	44,7	116,1	238,1	95,1	72,5	25,3	612,5
Maximum Value	22,0	44,8	116,6	239,9	96,0	73,0	25,4	615,3