1. CREDIT RISK. Ratings. Default probability. Risk premium. Recovery Rate

Credit risk arises from the variability of future returns, values, cash flows, earnings and other stated goals caused by changes in credit quality (creditworthiness) including default of the issuer, the borrower or the seller. Anyone who invests, lends or buys financial assets is exposed to credit risk. The credit quality is in fact a set of qualities which determine the ability of the borrower to meet his financial obligations and in particular to repay his debt. The credit quality is a qualitative attribute of a borrower although many qualitative and quantitative factors are used in credit analysis.

Credit risk involves changes in goals caused by upgrades and downgrades in credit quality. Such approach allows to calculate the diversification benefits and assess VaR.

The default risk is defined as the risk that the borrower will fail to satisfy the terms of his contractual obligation, such as repayment of the loan or payment of interests. The default of a borrower is the most unpleasant change of his credit quality for a lender.

Credit spread risk is the risk that the credit spread for a bond or a loan will increase. The U.S. Treasury bills and bonds are considered to be without credit risk. The corporate bonds and loans are priced at a spread to risk-free interest rates. Credit spread (premium, margin) should compensate for the credit risk associated with a bond or loan. Declining credit quality results in the widening in credit spread and in the same time in the fall in value of financial asset (bond, loan).

Downgrade risk occurs when a rating agency reduces its outstanding credit rating for an issuer or an issue. A deterioration in the credit quality of a borrower or an issue is penalized by the assignment of an inferior credit rating.

In general credit quality may be measured by

- subjective opinion by the lender (bank, corporation) or the more objective opinion issued by the rating agency,
- subjective credit margin established by a bank or objective market credit spread,
- subjective probability of default or objective market probability of default.

Fundamental Methods of Credit Quality Analysis

Fundamental analysis techniques are the most traditional and still very popular methods used to examine credit quality. These methods are used by banks, rating agencies and corporations. The C's of credit include

1. character,
2. corporate governance,
3. communication,
4. capacity,
5. capital,
6. collateral,
7. covenants,
8. composition,
9. concentration,
10. conditions.

Corporate Governance

Capacity

Capacity is the ability of the borrower to repay its obligations. Discretionary cash flow (operating cash flow less nondiscretionary capital expenditures) is the cash flow available to a company after if has funded its basic operating and capex requirements. Discretionary cash flow analysis is an important tool in the capacity analysis.

$$\frac{\text{EBITDA}}{\text{Gross Interest Expense}} > 3 \tag{1}$$

Capital

Capital (economic capital) should be sufficient to cover losses. Capital structure (Debt : Equity) determines default probability.

$$\frac{\text{Total Net Debt}}{\text{EBITDA}} < 3 \tag{2}$$

Collateral

Collateral is traditionally defined as assets that secure a loan or other debt, so that the assets may be seized by the lender if the borrower fails to make proper payments on the loan. The useful life of the collateral will typically have to exceed, or at least meet, the term of the loan. In broader sense collateral includes not only pledged assets, but also the quality and the value of unpledged assets owned or controlled by the issuer.

Covenants

Covenants are the terms and conditions of the lending agreement protecting the interests of a lender. There are various kinds of limitations and restrictions. Rate covenants impose obligation to maintain the specified rates. Affirmative covenants force a borrower to do something. Negative covenants forbid the specified activities. Protective covenants require additional actions for example to maintain an insurance. Covenants prevent the value transfer from debt holders to equity holders. A breach of any covenant provides an early warning to take action before the situation deteriorates further.

Composition

Composition or organizational structure of a borrower may be complex. It is important to look into the holding structure and predict how cash may flow between the parent company and subsidiaries. The assets of subsidiaries may be used as a collateral for a holding company.

Restricted subsidiaries are those considered to be consolidated for financial test purposes. Unrestricted subsidiaries are excluded from covenants and are not restricted to use new loans.

Conditions

Conditions in the branch, the industry, the national economy and the global economy have huge effects on a borrower's credit quality. The previously discussed factors represent the unique factors and are part of the unique credit risk. The conditions determine the systematic risk.

Credit ratings

The most known rating agencies in the world are: Standard & Poor's Corporation, Moody's Investors Service Inc. and Fitch.

Credit rating agencies provide opinions on the creditworthiness of issuers of securities and other financial obligations. They rate issues of securities, corporate and governmental issuers and structured financings. They also assess the credit quality of financial guarantees, bank loans, private placements, MBS (mortgage-backed securities) and ABS (asset-backed securities), mutual funds and the ability of insurance companies to pay claims. The fundamental premises of the rating agencies are: integrity, independence, objectivity, transparency, credibility and quality of their opinions and operations.

A credit rating is the opinion of the creditworthiness of an obligor, or the creditworthiness of an obligor with respect to a particular debt security or other financial obligation, based on relevant risk factors. Credit ratings that may apply to an issuer's general creditworthiness or to a specific financial obligation. A rating does not form a recommendation to purchase, sell, or hold a particular security.

Table 1. Credit Ratings

Credit Quality	S&P's	Moody's
Investment grades		
The obligor's capacity to meet its financial commitment on the obligation is extremely strong.	AAA	Aaa
The obligor's capacity to meet its financial commitment on the obligation is very strong.	AA	Aa
An obligation is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than obligations in higher rated categories. However, the obligor's capacity to meet its financial commitment on the obligation is still strong.	A	A
Adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitment on the obligation. An obligation rated 'BBB' exhibits adequate protection parameters.	BBB	Baa
Speculative grades		
Inadequate capacity to meet financial commitment on the obligation	BB	Ba
The obligor's capacity or willingness to meet its financial commitment on the obligation is likely impair	B	B
The obligor is not likely to have the capacity to meet its financial commitment on the obligation in the event of adverse business, financial, or economic conditions.	CCC	Caa
A default has actually occurred.	D	D

Source: Standard and Poor's and Moody's Investors Service

18 categories

SnP18	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
Moody18	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3	Caa	D

Default probability

Credit scoring

Credit scoring models use statistical date to calculate probability of default or to sort borrowers into default risk classes. For consumer loans the characteristics in credit scoring model might include age, income, assets, location etc. For corporate loans financial ratios are used as variables. A statistical model quantifies or scores the default risk probability. Some credit scoring techniques use probability models, discriminative models and other econometric models.

Probability Model, Logit, Probit

Linear Probability Model

$$(3) \qquad Z_i = \sum_{j=1}^{n} \beta_j X_{ij} + \xi_j$$

The Logit Model constraints the estimated probability of default to lie between zero and one. It calculates the cumulative probability of default according to the formula

$$(4) \qquad F(Z_i) = \frac{1}{1 + e^{-Z_i}}$$

where
Z_i – is estimated by regression model similar to the linear probability model.

The probit model also constraints the estimated probability of default to lie between zero and one. The cumulative normal distribution function is used to calculate estimated probability of default.

Mortality Rates

Table 2. Adjusted Mortality Rates according to Standard & Poor's (1971-1988)

Grupa		1	2	3	4	5	6	7	8	9	10
AAA	Roczne	0,00%	0,00%	0,00%	0,00%	0,00%	0,15%	0,05%	0,00%	0,00%	0,00%
	Skumulowane	0,00%	0,00%	0,00%	0,00%	0,00%	0,15%	0,21%	0,21%	0,21%	0,21%
AA	Roczne	0,00%	0,00%	1,39%	0,33%	0,20%	0,00%	0,27%	0,00%	0,11%	0,13%
	Skumulowane	0,00%	0,00%	1,39%	1,72%	1,92%	1,92%	2,18%	2,18%	2,29%	2,42%
A	Roczne	0,00%	0,39%	0,32%	0,00%	0,00%	0,11%	0,11%	0,07%	0,13%	0,00%
	Skumulowane	0,00%	0,39%	0,71%	0,71%	0,71%	0,82%	0,93%	1,00%	1,13%	1,13%
BBB	Roczne	0,03%	0,20%	0,12%	0,26%	0,39%	0,00%	0,14%	0,00%	0,21%	0,80%
	Skumulowane	0,03%	0,23%	0,35%	0,61%	1,00%	1,00%	1,14%	1,14%	1,34%	2,13%
BB	Roczne	0,00%	0,50%	0,57%	0,26%	0,53%	2,79%	3,03%	0,00%	0,00%	3,48%
	Skumulowane	0,00%	0,50%	1,07%	1,34%	1,86%	4,59%	7,48%	7,48%	7,48%	10,70%
B	Roczne	1,40%	0,65%	2,73%	3,70%	3,59%	3,86%	6,30%	3,31%	6,84%	3,70%
	Skumulowane	1,40%	2,04%	4,72%	8,24%	11,54%	14,95%	20,31%	22,95%	28,22%	30,88%
CCC	Roczne	1,97%	1,88%	4,37%	16,35%	2,06%	0,00%	0,00%	0,00%	0,00%	0,00%
	Skumulowane	1,97%	3,81%	8,01%	23,05%	24,64%	24,64%	24,64%	24,64%	24,64%	24,64%

Źródło: E.I.Altman, Default Risk, Mortality Rates, and the Performance of Corporate Bonds, ICFA, 1989

Linear Discriminant Model

The most famous Altman's discriminant model is[1]:

(5) $Z = 1{,}2\ X_1 + 1{,}4\ X_2 + 3{,}3\ X_3 + 0{,}6\ X_4 + 0{,}999\ X_5$

where
X_1 – working capital / total assets,
X_2 – retained earnings / total assets,
X_3 - EBIT / total assets,
X_4 – market value of equity / book value of long term debt,
X_5 - sales / total assets.

Market Probability of Default

The implied probability of repayment can be calculated using spot or forward rates for a corporate security and government securities. The cumulative probability of repayment can be calculated using formula:

(6) $$p_{wsT} = \frac{(1+R_B)^T}{(1+R_K)^T}$$

where
p_{wsT} – cumulative default probability in horizon T,
R_B – spot risk-free interest rate,
R_K – spot rate for a corporate security
T - maturity.

Migration matrix

Table 3. Migration Matrix (Moody's)

2001	Aaa	Aa	A	Baa	Ba	B	Caa	D	WR
Aaa	89,91	0,92	0,00	0,00	0,00	0,00	0,00	0,00	9,17
Aa	0,30	90,92	5,06	0,15	0,00	0,15	0,00	0,00	3,42
A	0,25	2,30	86,69	6,57	0,33	0,16	0,16	0,16	3,38
Baa	0,19	0,29	3,58	86,83	4,16	0,77	0,19	0,29	3,70
Ba	0,00	0,00	1,16	6,55	75,53	8,86	1,16	1,16	5,58
B	0,00	0,00	0,11	0,95	5,58	63,44	14,54	9,17	6,21
Caa-C	0,00	0,00	0,00	0,00	0,00	2,34	54,69	30,47	12,50

Source: Moody's Investors Service, Global Credit Research, February 2002

Distance to default

Distance to default default point) may be calculalated:

(7) $$d = \frac{S - S_d}{\sigma_{ROA} S}$$

where
S – market value of assets (Merton model),
S_d – default point (1/2 of long term debt + current liabilities),
σ_{ROA} - volatility of the firm's assets.

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[1] Por. E.I. Altman, *Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy, Journal of Finance*, September 1968, s. 189-209 oraz E.I. Altman, *Managing the Commercial Lending Process*, w: *Handbook of Banking Strategy*, John Wiley, New York 1985, s. 473-510.

Risk premium

Market risk premium

Risk premium may be defined as

(8) $$q = R_K - R_B$$

where
p_{wsT} – cumulative default probability in horizon T,
R_B – spot risk-free interest rate,
R_K – spot rate for a corporate security
T - maturity.

More realistic situation is that the lender does not lose all the principal. The recovery rate u depends on the type of collateral. We can use the following formula to establish the appropriate risk premium

(9) $$1 + R_B = p_{wsT}(1 + R_O) + (1 - p_{wsT})(1 + R_O)\,u$$

where
u – recovery rate

Risk premium is equal to:

(10) $$q = R_O - R_B = \frac{(1 + R_B)}{p_{wsT} + u - p_{wsT}u} - (1 + R_B)$$

Risk premium based on BSM model

Problem 1. Spot Position. Forward position

An option model application

S - market value of assets	4 756	PLN thousands
E - book value of debt	5 000	PLN thousands
t - maturity of debt	360	days
R_B^* - risk free rate	5,0%	
σ^2 - volatility (ROA)	20%	

(a) Calculate the economic value of debt and equity.
(b) Calculate the risk premium. Show the sensitivity of risk premium to maturity (90, 180, 360 days) and leverage ratio (0,6, 0,7, 0,8, 0,9, 1).
(c) Show the sensitivity of risk premium to volatility (5%,10%,15%,20%,25%,30%) and leverage ratio (0,6, 0,7, 0,8, 0,9, 1).

Solution

(a)

The economic value of debt $S - C = Ee^{-R_B^* T}\left[\frac{1}{d}N(-d_1) + N(d_2)\right]$

maturity	$T =$	1,00
continuously compunded discount factor	$e^{-R_B^* T} =$	0,9512
present value of debt	$Ee^{-R_B^* T} =$	4 756,1
leverage ratio $d = \frac{Ee^{-R_B^* T}}{S}$		1,00

auxiliary variables d_1 and d_2 :

$d_1 = -\frac{\ln(d) - \frac{1}{2}\sigma_s^2 T}{\sigma_s\sqrt{T}} =$ 0,1000 $\quad d_2 = -\frac{\ln(d) + \frac{1}{2}\sigma_s^2 T}{\sigma_s\sqrt{T}} =$ -0,1000

Cumulative probabilities

$N(-d_1) =$ 0,4602 $\quad N(d_2) =$ 0,4602

Cumulative probability of repayment

$\left(\frac{1}{d}N(-d_1) + N(d_2)\right) =$ 0,9203

The economic value of debt $S - C = Ee^{-R_B^* T}\left[\frac{1}{d}N(-d_1) + N(d_2)\right] =$ 4377,3

The economic value of equity $C =$ 378,9

(b)

Risk premium is $q^* = -\frac{1}{T}\ln\left[\frac{1}{d}N(-d_1) + N(d_2)\right]$ = q^* = 8,30%

Risk premium sensitivity

	Risk premium			Probability of repayment		
8,30%	90	180	360	90	180	360
0,6	0,00%	0,00%	0,04%	1,0000	1,0000	0,9996
0,7	0,00%	0,06%	0,36%	1,0000	0,9997	0,9965
0,8	0,20%	0,77%	1,49%	0,9995	0,9961	0,9852
0,9	3,18%	3,98%	4,07%	0,9921	0,9803	0,9601
1	16,28%	11,60%	8,30%	0,9601	0,9436	0,9203

(c)

Risk premium sensitivity

Risk premium

8,30%	5%	10,00%	15,00%	20,00%	25,00%	30,00%
0,6	0,00%	0,00%	0,00%	0,04%	0,24%	0,70%
0,7	0,00%	0,00%	0,05%	0,36%	1,03%	2,06%
0,8	0,00%	0,05%	0,51%	1,49%	2,87%	4,52%
0,9	0,03%	0,79%	2,27%	4,07%	6,04%	8,11%
1	2,01%	4,07%	6,16%	8,30%	10,48%	12,70%

Probability of repayment

0,6	1,0000	1,0000	1,0000	0,9996	0,9976	0,9930
0,7	1,0000	1,0000	0,9995	0,9965	0,9897	0,9796
0,8	1,0000	0,9995	0,9950	0,9852	0,9717	0,9558
0,9	0,9997	0,9921	0,9775	0,9601	0,9414	0,9221
1	0,9801	0,9601	0,9402	0,9203	0,9005	0,8808