Risk Management using Options. Hedge ratios

Hedge ratio

Suppose an investor has a portfolio consisting of one unit of asset 1 and h units of asset 2, that is:

$V = I_1 + hI_2$

The change in the value of the portfolio as the underlying asset price changes is

$$\frac{\partial V}{\partial S} = \frac{\partial I_1}{\partial S} + h\frac{\partial I_2}{\partial S}$$

or in terms of delta

$$\Delta_v = \Delta_1 + h\Delta_2$$

The hedge ratio, h, to obtain desired delta for the portfolio is equal to:

$$h = \frac{\Delta_v - \Delta_1}{\Delta_2}$$

An investor should hold h units of asset 2 for every unit of asset 1.

Delta-neutral hedge

Delta neutral hedge position is constructed using underlying assets and derivatives in the way that the net delta for the combined position is equal zero. It means that the value of a portfolio does not change as the underlying asset price changes. For a complete or delta-neutral hedge (Δ_v=0) the hedge ratio would be

$$h = \frac{-\Delta_1}{\Delta_2}$$

The relationship between change in call option price and sensitivity measures

$$\Delta C = (\Delta_c + \frac{1}{2}\gamma_c \Delta S)\Delta S - \theta_c \Delta T + \rho_c \Delta r + \upsilon_c \Delta\sigma$$

where
ΔC – change in call option price,
ΔS – change in underlying asset price,
ΔT – change in time to expiration,
Δr – change in interest rates,
$\Delta\sigma$ – change in volatility,
Δ_c - delta,
γ_c - gamma,
θ_c - theta,
ρ_c - rho,
υ_c - vega.

Synthetic positions using the put/call parity

A synthetic underlying asset. This position can be replicated by buying a call option, selling a put option and buying a riskless Treasury Bill.

$$(1)\, S_0 = C - P + \frac{E}{1+rT}$$

A synthetic call option. Buying an underlying asset, buying a put, and borrowing money at riskless interest rate can replicate the synthetic long call position.

$$(2)\, C = S_0 + P - \frac{E}{1+rT}$$

A synthetic put option. The put option position can be thought as being equal to buying a call, a short position in the underlying asset, and buying a riskless Treasury Bill.

$$(3)\, P = C - S_0 + \frac{E}{1+rT}$$

A synthetic Treasury Bill. The payoff of a Treasury Bill can be replicated buy buying an underlying asset, buying a put option and selling a call option.

$$(4)\, \frac{E}{1+rT} = S_0 + P - C$$

A synthetic covered call. This strategy may be constructed by selling a put option and buying the riskless Treasury Bill.

$$(5)\, S_0 - C = -P + \frac{E}{1+rT}$$

A synthetic protective put. The protective put strategy may be created by buying a call option and buying a Treasury Bill.

$$(6)\, S_0 + P = +C + \frac{E}{1+rT}$$

This strategy is sometimes referred to as 90/10, because an investor uses approximately 90 percent of his money to buy a riskless Treasury Bill and 10 percent to buy call options.

Dynamic Option Replication

Instead of buying call options or put options an investor may replicate these positions by frequently trading the underlying security and a riskless Treasury bill. The proportion of money put with the underlying asset and proportion put with the Treasury bill are dynamically change as the sensitivity measures of appropriate option positions change.

Problem 1. Hedge Ratio

An investor wants to hedge a position in the underlying asset with options.
Suppose the delta of a put option is -0,40.
(a) What position in a put option would create a delta-neutral hedge?
(b) What would be the required put position, if the desired delta for the hedged position were 0,50?
(c) What position using call options would give the same delta to that in the question above?

Solution

(a)

The hedge ratio is calculated as

$$h = \frac{\Delta_v - \Delta_1}{\Delta_2}$$

To create a delta-neutral hedge for a portfolio using a put option, a hedge ratio should be (0 - 1) : (-0,40) = 2,5

An investor should buy 2,5 put options for a unit of an underlying asset.

(b)

To create a delta of 0,50 for a portfolio, the hedge ratio should be (0,50 - 1) : (-0,40) = 1,3

An investor should buy 1,3 put options for a unit of an underlying asset.

(c)

The delta of the equivalent call option is :

$\Delta C = \Delta P + 1 =$ 0,6

To create a delta of 0,50 for a portfolio, the hedge ratio should be (0,50 - 1) : (0,60) = -0,8

An investor should sell 0,8 call options for a unit of an underlying asset.

Problem 2. A Delta/Vega – Neutral Position

An investor wants to hedge an underlying asset using a call and a put options with the following parameters.

	Call	Put
S = spot price	100	100
E = exercise price	90	95
Time to expiration (number of days)	30	30
Risk free interest rate	5%	5%
σ = volatility	22%	30%
q - dividends yield		

(a) Calculate delta and vega for a call option and a put option.
(b) Calculate the hedge ratios required to construct a delta/vega-neutral position using the underlying asset, a call option and a put option.

Solution

(a)

	Underlying asset	Call option	Put option
Delta	1	0,96	-0,25
Vega	0	0,02	0,09

(b)

The determinant of the matrix 0,092738

$A * h = b$ so $h = A^{-1} * b$

The inverse matrix is

$$\begin{bmatrix} 0{,}97 & 2{,}65 \\ -0{,}26 & 10{,}37 \end{bmatrix} * \begin{bmatrix} -1{,}0000 \\ 0{,}0000 \end{bmatrix} = \begin{bmatrix} h_1 & 1 \\ h_2 & -0{,}9739 \\ h_3 & 0{,}2588 \end{bmatrix}$$