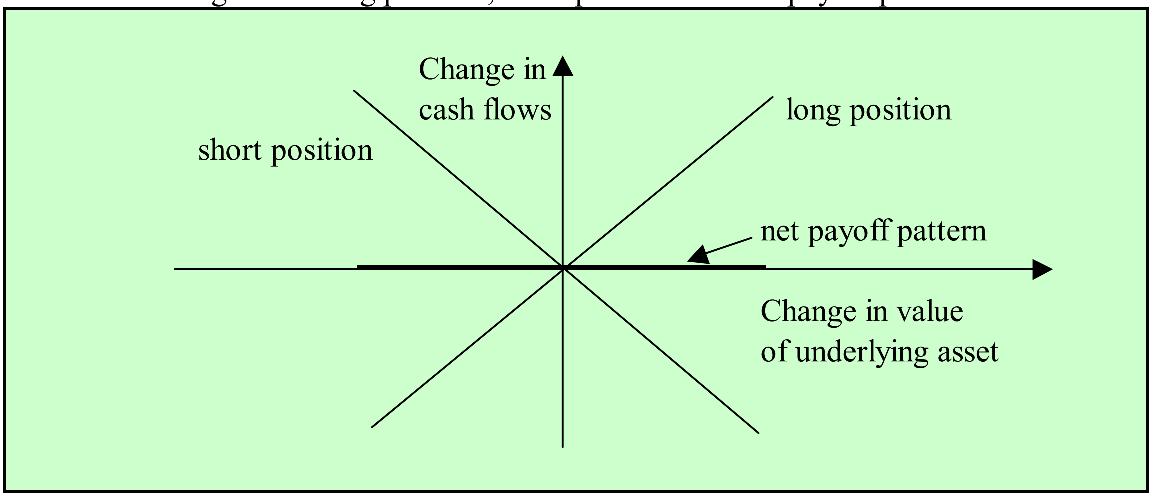
Risk Management with forward and futures

1.1 Long position and short position

A firm is said to take a long position (have a long exposure) in an asset when it currently holds the asset or will receive the asset in the future. Under a long position, there is a direct relationship between changes in the value of the asset and changes in the firm's value.

A firm assumes a short position in an asset when it is obliged to give up the asset in the future or must obtain the asset in the future at an unknown price. In short position, there is an inverse relationship between the value of the firm and the value of the asset held short.

Figure 1. Long position, short position and net payoff pattern



A futures contract to **purchase** an asset produces a **long position**, with the value of the futures contract increasing as the value of the underlying asset increases. A futures contract to **sell** an asset produces a **short position**, with the value of the futures contract decreasing as the value of the underlying asset increases. The payoff diagrams for long and short positions in futures are identical to the payoff diagrams for forwards.

1.2 Value of the hedged position prior to expiration

Two types of hedging:

- *inventory hedge* - investor holds an asset and sells a futures contract to hedge its price movement,
- *anticipatory hedge* - investor has a short position in an asset and purchases a futures contract.

The value of the hedged position at time t is equal to the current market price of the underlying asset plus the gain or loss on the futures contract $(F - F_t)$[1]:

(1) $V_t = S_t + (F - F_t)$

or

(2) $V_t = F - (F_t - S_t)$

or

(3) $V_t = S + [(F - S) - (F_t - S_t)]$

The value of the hedged position is equal to:

- market price of the underlying asset plus the gain or loss on the futures contract,
- initial futures price plus basis,
- market price plus change in basis.

The value of the hedged position at expiration should be F. The basis is generally zero $S_t=F_t$. A hedged position reduces the price risk in the underlying asset to just the risk in the basis.

[1] The formulas below are for inventory hedge. For anticipatory hedge we have to change signs.

1.3 Hedge Ratio Alternatives

Risk management with forward and futures contracts use hedge ratios which indicate what percentage of the price movement in underlying asset or gap should be covered by forward or futures. Table 1 lists the techniques used to establish hedge ratios.

Table 1. Hedge Ration Techniques

Equal-dollar match	h=-1
Theoretical	h=- ΔS/ΔF
Minimum variance	$h=\frac{-\rho_{SF}\sigma_{\Delta S}}{\sigma_{\Delta F}}$
Statistical estimation	h = negative of the slope coefficient of regression of ΔF on ΔS

The number of futures contracts required is calculated using the formula

(4) $n = \frac{hK}{k}$

where:
K – hedge value
k – contract size.

1.3.1 The Equal-dollar-matched Hedge Ratio

The equal-dollar-matched hedge ratio is 1 for long and –1 for short position in forward or futures. These ratios may not create the optimal hedge.

1.3.2 Theoretical Hedge Ratios

Theoretical hedge ratios apply the relationship between the forward price and the spot price. Assume that the hedged position is formed by holding the underlying asset and selling h futures contracts

(5) $V_t = S_t - h\,(F - F_t)$
and
(6) $V_{t+1} = S_{t+1} - h\,(F - F_{t+1})$
where:
S_t – the spot price,
F_t – the futures price,
h – hedge ratio (negative for the short).

The change in the value of the hedged position is:

(7) $\Delta V = \Delta S + h\Delta F$

The hedge ratio would be

(8) $h = \frac{\Delta V - \Delta S}{\Delta F}$

For a theoretical, or delta nautral hedge ΔV =0 and the hedge ratio is

(9) $h = \frac{-\Delta S}{\Delta F}$

1.3.3 The Minimum Variance Hedge Ratio

The hedge ratio which minimizes the variance of the hedged value is referred to as the minimum-variance hedge ratio. Using (7) we may derive:

(10) $\sigma^2_{\Delta V} = \sigma^2_{\Delta S} + h^2\sigma^2_{\Delta F} + 2h\rho_{SF}\sigma_{\Delta S}\sigma_{\Delta F}$

where:

$\sigma^2_{\Delta V}, \sigma^2_{\Delta F}, \sigma^2_{\Delta S}$ - variance of the hedged position, futures prices and spot prices,

ρ_{SF} - correlation coefficient between futures prices and spot prices.

The minimum variance hedge ratio is:

(11) $h = \dfrac{-\rho_{SF}\sigma_{\Delta S}}{\sigma_{\Delta F}}$

1.3.4 Statistical Estimation

The hedge ratio can be calculated by regressing the change in the futures price on the change in the underlying asset price.

(12) $\Delta S = \alpha + \beta\Delta F + \varepsilon$

where

α, β - parameters, ε -residual error.

The hedge ratio is

(13) $h = - b = \dfrac{-\rho_{SF}\sigma_{\Delta S}}{\sigma_{\Delta F}}$

1.4 Theoretical Hedge Ratios

Theoretical hedge ratios can be calculated using the arbitrage relationship between the fair price of futures contract and the spot price of the underlying security.

1.4.1 Equity Index Futures

The change in the price of an equity portfolio depends on a factor of beta.

(14) $\Delta S = \beta\Delta I$

where

ΔS – the change in the spot price, β - beta coefficient,

ΔI – the change in the market index.

The price change in the futures contract is given by:

(15) $\Delta F = (1 + \dfrac{rt}{360})\Delta I$

The hedge ratio for an equity portfolio can be calculated as:

(16) $h = \dfrac{-\Delta S}{\Delta F} = \dfrac{-\beta}{(1 + \dfrac{rt}{360})}$

1.4.2 Currency futures

The change in the futures contract's price relative to the spot exchange rate is

(17) $$\Delta F = \Delta S_0 \frac{\left(1 + i_d^N T\right)}{\left(1 + i_f^N T\right)}$$

The hedge ratio can be calculated as:

(18) $$h = -\frac{\Delta S_0}{\Delta F} = -\frac{\left(1 + i_f^N T\right)}{\left(1 + i_d^N T\right)}$$

1.4.3 Eurodollar Futures

For an Eurodollar deposit the change in price from a change in money market yield is:

(19) $$\Delta S = \frac{-\left(\frac{t}{360}\right)\Delta r}{1 + \frac{rt}{360}}$$

The change in the price of an eurodollar futures is:

(20) $$\Delta F = -0{,}25\Delta r_f$$

The hedge ratio can be calculated as:

(21) $$h = \frac{-\Delta S}{\Delta F} = \frac{\left(-\frac{t}{360}\right)}{0{,}25\left(1 + \frac{rt}{360}\right)}\left(\frac{\Delta r}{\Delta r_f}\right)$$

$\frac{\Delta r}{\Delta r_f}$ is the relative movement in the interest rates.

1.4.4 Treasury Bill Futures

The change in spot price is

(22) $$\Delta S = -\left(\frac{t}{360}\right)\Delta d$$

where
Δd – the change in the discount rate on the Treasury bill

The change in the price of the futures contract is:

(23) $$\Delta F = 0{,}25\Delta d_f$$

The hedge ratio can be calculated as:

(24) $$h = \frac{-\Delta S}{\Delta F} = \frac{\left(-\frac{t}{360}\right)}{0{,}25}\left(\frac{\Delta d}{\Delta d_f}\right)$$

1.4.5 Bond Futures

The change in the value of a bond is:

(25) $\Delta P = -D_O P y_O$

where
D_O – the modified duration of the bond,
P – the bond price,
y_O - YTM.

The change in the value of the bond futures contract is:

(26) $\Delta F = -D_F F y_F$

D_F - the modified duration of the futures contract,
F – the futures price,
y_F – YTM of the CTD.

The hedge ratio is:

(27) $$h = \frac{-\Delta S}{\Delta F} = \frac{-D_O P}{D_F F}\left(\frac{\Delta y_O}{\Delta y_F}\right)$$

The modified duration of the futures contract can be calculated using the formula

(28) $$D_F = -\frac{1}{F}\left(\frac{dF}{dy}\right) = \frac{P}{fF}\left[D\left(1+\frac{rt}{360}\right) - \left(\frac{dr}{dy}\right)\frac{t}{360}\right]$$

1.4.6 Stock and Bond Portfolio

Forward and futures contract allow to change the structure and risk characteristics of the assets. Suppose an investor wants the stocks value to change as if u_A^p were invested in equity with β_A^p sensitivity to the market index. The current proportion is u_A with β_A risk. The hedge ratio required to give the desired effects is:

(29) $$h_A = \frac{u_A^p \beta_A^p - u_A \beta_A}{\left(1+\frac{rt}{360}\right)}$$

Suppose an investor wants the bonds value to change as if u_O^p were invested in bonds with the modified duration D_O^p. The current proportion is u_O with D_O risk. The hedge ratio required to give the desired effects is:

(30) $$h_O = \frac{P(u_O^p D_O^p - u_O D_O)}{D_F F}\left(\frac{\Delta y_O}{\Delta y_F}\right)$$

Problem 1. The equal-dollar-matched hedge ratio

An investor will receive a payment of $1 million from a major export contract in 30 days. The current spot exchange rate is 4,5709 PLN/$, but investor is concerned that the dollar will depreciate. The dollar futures contract is priced now at 4,6400 PLN/$.

(a) How many contracts would need to be used to hedge the decline in value assuming that the contract size is $10000.

(b) If the spot exchange rate subsequently declines to 3,8000 PLN/$ and the futures price declines to 3,8603 PLN/$, what is the net result of the hedge for the investor.

Solution

(a)

If the zloty value of the dollar falls, the anticipated payment in zloties will be worth less in zloties. To hedge against currency risk, the investor would need to sell 100 futures contracts. The hedge ratio is h = -1.

$$n = \frac{hK}{k} \qquad = \qquad 100 \text{ contracts}$$

(b)

	Exchange rate	Value $ millions	PLN millions
Spot exchange rate t=0	4,5709	1,000	4,5709
Furtures exchange rate t=0	4,6400	1,000	4,6400
Spot exchange rate t=1	3,8000	1,000	3,8000
Futures exchange rate t=1	3,8603	1,000	-3,8603
Value of the hedged position			4,5797

The value of the hedged position at time t is equal to the initial futures price plus the basis at time t:

$(F + (S_t - F_t)) * K =$ (4,6400 + 3,8000- 3,8603) * 1 = 4,5797 PLN million.

The value of the hedged position would be the same as the original forward position if the basis had closed to zero at the termination of the futures contract.

Problem 2. Theoretical hedge ratio. Equity hedge

An investor owns a PLN 1 million equity porfolio that has a beta of 1,5.
The index futures expire in 73 days and the current risk-free interest rate is 18,00%.
The current WIG 20 index is 1662,2. The contract size is PLN 16622.
(a) Calculate a delta neutral hedge ratio.
(b) Calculate the number of contracts required to hedge the position.

Solution

(a)

The hedge ratio for the equity porfolio is $h = \frac{-\Delta S}{\Delta F} = \frac{-\beta}{(1+\frac{rt}{360})}$ = -1,45

(b)

$$n = \frac{hK}{k} = -87{,}064$$

The equity portfolio should be hedged by selling 87 index futures contracts.

Problem 3. Theoretical hedge ratio. Currency hedge

An investor wants to hedge a $1 million currency gap against currency risk.
The currency futures expire in 78 days. The current home risk-free interest rate is 18,0%, and foreign risk-free interest rates is 6,0%. The contract size is $ 10 000.
(a) Calculate a delta neutral hedge ratio.
(b) Calculate the number of contracts required to hedge the position.

Solution

(a)

The hedge ratio for the equity porfolio is $h = -\frac{\Delta S_0}{\Delta F} = -\frac{\left(1+i_f^N T\right)}{\left(1+i_d^N T\right)}$ = -0,975

(b)

$$n = \frac{hK}{k} = -97{,}5$$

The currency gap should be hedged by selling 97 currency futures contracts.

Problem 4. Theoretical hedge ratio. Eurodollar hedge

An investor wants to hedge a Eurodollar deposit $4 million with 45 days to maturity. The risk-free interest rate is 5,4%. The contract size is 1 000 000 USD.

(a) Calculate a delta neutral hedge ratio assuming that $\frac{\Delta r}{\Delta r_f} = 1$.

(b) Calculate the number of contracts required to hedge the position.

Solution

(a)

The hedge ratio is $h = \frac{-\Delta S}{\Delta F} = \frac{\left(-\frac{t}{360}\right)}{0{,}25\left(1 + \frac{rt}{360}\right)}\left(\frac{\Delta r}{\Delta r_f}\right) =$ -0,497

(b)

$$n = \frac{hK}{k} = -1{,}98659$$

The hedge position requires only 2 two eurodollar futures contracts, because deposit has only 45 days of interest exposure left and futures contracts expire in 90 days.

Problem 5. Theoretical hedge ratio. Treasury bill hedge

An investor wants to hedge $ 10 million Treasury bills with 37 days to maturity. The futures contract size is $1 000 000.

(a) Calculate a delta neutral hedge ratio assuming that $\frac{\Delta d}{\Delta d_f} = 1$

(b) Calculate the number of contracts required to hedge the position.

Solution

(a)

The hedge ratio is $h = \frac{-\Delta S}{\Delta F} = \frac{\left(-\frac{t}{360}\right)}{0{,}25}\left(\frac{\Delta d}{\Delta d_f}\right) =$ -0,41

(b)

$$n = \frac{hK}{k} = -4{,}11$$

An investor should sell 4 futures contracts.

Problem 6. Theoretical hedge ratio. Bond hedge

An investor wants to hedge \$1 million Treasury bonds. The security price is \$93. 5/32. The futures price is 78. 24/32. The modified duration of the security and the futures contract are 7 years and 7 years, respectively. The contract size is \$100 000.

(a) Calculate a delta neutral hedge ratio assuming that $\frac{\Delta y_O}{\Delta y_F} = 1$

(b) Calculate the number of contracts required to hedge the position.

Solution

(a)

The hedge ratio is $h = \frac{-\Delta S}{\Delta F} = \frac{-D_O P}{D_F F}\left(\frac{\Delta y_O}{\Delta y_F}\right)$ = -1,19

(b)

$n = \frac{hK}{k}$ = -11,9

An investor should sell 12 futures contracts.

Problem 7. Theoretical hedge ratio. Portfolio hedge

An investor wants to hedge a portfolio $10 million of stocks and bonds.
The current and desired characteristics are shown below.

Portfolio	Underlying	Desired
Stocks	3 000 000	8 000 000
Bonds	7 000 000	2 000 000
Stocks in percentage	30,0%	80,0%
Bonds in percentage	70,0%	20,0%
Beta (stocks)	0,10	1,50
Duration (bonds)	5,00	8,00

Index futures contract:
expiration 44 days, risk-free rate 12,3%,
current index 1 485,3, contract size $14 853.

Bond futures:
maturity 90 days, spot price 93. 5/32, futures price 78. 24/32,
modified duration of the futures contract 7,35 years,
price of CTD bond 124. 6/32, size of the futures contract $124 202 .

(a) Calculate a delta neutral hedge ratio for the desired equity exposure.
(b) Calculate the number of contracts required to hedge the position.
(c) Calculate a delta neutral hedge ratio for the desired bonds exposure.
(d) Calculate the number of contracts required to hedge the position.

Solution

(a)

The hedge ratio is

$$h_A = \frac{u_A^p \beta_A^p - u_A \beta_A}{\left(1 + \frac{rt}{360}\right)} \qquad = \qquad 1,153$$

(b)

$$n = \frac{h_A u_A^p K}{k_A} \qquad = \qquad 620,8$$

An investor should buy 621 index futures contracts.

(c)

The hedge ratio is

$$h_O = \frac{P(u_O^p D_O^p - u_O D_O)}{D_F F}\left(\frac{\Delta y_O}{\Delta y_F}\right) \qquad = \qquad -0,306$$

(d)

$$n = \frac{h_O u_O^p K}{k_O} \qquad = \qquad -4,9$$

An investor should sell 5 bond futures contracts.