Pricing Interest Rate and currency Swaps. Up-front fee. Valuation (MTM)

A plain vanilla swap pricing is the process of setting the fixed rate, so that the initial value of the swap is zero for both counterparties. Thereafter it is positive for one counterparty making it an asset and negative for another counterparty making it a liability. It is found by comparing the agreed fixed rate to the current fixed rate on a swap having the same maturity as the original agreement. The process of valuation is called "mark-to-market".

A plain vanilla interest swap's rate is its fixed rate. Although the swap fixed rate is quoted off the Treasury yield curve, it is priced off an appropriate forward curve corresponding to the floating reference rate on the swap.

The swap's fixed rate should be established at the level so that the present value of the fixed cash flows eventually adjusted by an up-front fee equals the present value of the floating cash flows implied by the forward rates. These forward rates may be observed, calculated or estimated. The appropriate spot rates are used as discounting rates.

Swap with a LIBOR/WIBOR as a reference rate

The FRA interest rates

In the United States the FRA prices are derived from the observed LIBOR forward curve (Eurodollar futures) because it indicates directly the levels of the floating interest rate that can be locked in by arbitrage transactions. In Poland instead WIBOR futures do no exist and the FRA prices may be derived only using implied forward rates, which are inferred from the WIBOR and WIBID spot interest rates and eventually interest rates observed in the FX swap market.

Swap with a tenor up to 2 years

In the United States and in Poland the fixed rate of the interest rate swap with a tenor of up to two years is established using the observed FRA interest rates.

Swap with a tenor between 2 and 10 years

In the United States the fixed rate of the interest rate swap with a tenor of between 2 and 10 years is established using the observed LIBOR forward curve (Eurodollar futures). In Poland WIBOR futures do not exist. The forward curve for WIBOR must be estimated off the forward curve for Treasury securities. The following procedure for a U.S. swap exceeding 10 years applies.

Swap with a tenor exceeding 10 years

In the United States the fixed rate of the interest rate swap with a tenor exceeding 10 years is established using estimated forward rates. These estimated LIBOR forward rates are predicted using the forward rates for Treasury securities and independently forecasted TED (Treasury Eurodollar Difference). The forward rates for Treasury securities are implied forward rates calculated using spot Treasury curve. The spot Treasury curve is sometimes directly observed but it is usually derived using the "bootstrapping" procedure.

Swap with a Treasury yield as a reference rate

Assuming that floating-rate payments are made on the basis of a/360 and fixed-rate payments are made on the basis of a/365 (other assumptions will be introduced subsequently)

$$\sum_{t=1}^{T}\frac{{}_{t}f_{t-1}\frac{p_t}{360}K_t}{\left(1+z_t\frac{p_t}{360}\right)^t}=\sum_{t=1}^{T}\frac{s_T\frac{p_t}{365}K_t}{\left(1+z_t\frac{p_t}{360}\right)^t}+PP_0$$

where
${}_{t}f_{t-1}$ - forward rate,
p_t – number of days in period t
K_t – notional principal in period t,
s_T – swap fixed rate,
z_t – spot rate (discounting rate,
PP_0 – up-front fee,
t – period (t=1,2,...,T),

Swap fixed rate is equal to:

$$s_T=\frac{\sum_{t=1}^{T}\frac{{}_{t}f_{t-1}\frac{p_t}{360}}{\left(1+z_t\frac{p_t}{360}\right)^t}K_t}{\sum_{t=1}^{T}\frac{\frac{p_t}{365}}{\left(1+z_t\frac{p_t}{360}\right)^t}K_t}$$

Swap with a WIBOR (LIBOR) rate

$$(1)\qquad \sum_{t=1}^{T}\frac{{}_{t}f_{t-1}\frac{d_t-d_{t-1}}{360}K_t}{1+z_t\frac{d_t}{360}}=\sum_{t=1}^{T}\frac{s_T\frac{d_t-d_{t-1}}{365}K_t}{1+z_t\frac{d_t}{360}}$$

Swap fixed rate is equal to:

$$s_T=\frac{\sum_{t=1}^{T}\frac{{}_{t}f_{t-1}\frac{d_t-d_{t-1}}{360}}{1+z_t\frac{d_t}{360}}K_t}{\sum_{t=1}^{T}\frac{\frac{d_t-d_{t-1}}{365}}{1+z_t\frac{d_t}{360}}K_t}$$

Problem 1. Pricing FRA rates using Eurodollar futures

Suppose today is: 20-11-98. Three-month LIBOR is 5,25%
The three-month forward rates for Eurodollar futures contracts are following:

Maturity	Rate
12-98	5,19%
03-99	4,86%
06-99	4,86%
09-99	4,89%
12-99	5,16%
03-00	5,02%
06-00	5,09%
09-00	5,15%

Required

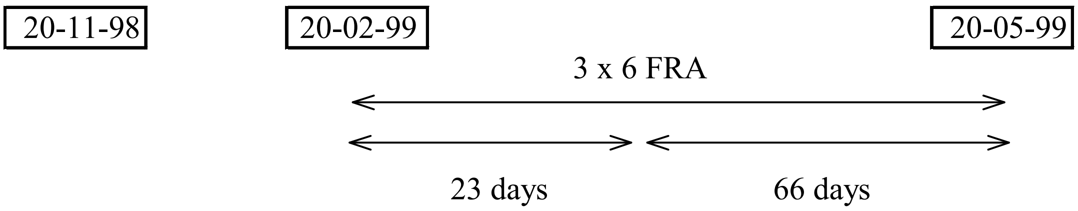
(a) Calculate the prices for a series 3 x 6, 6 x 9, 9 x 12, 12 x 15, 15 x 18, 18 x 21, 21 x 24 of FRA transactions based on Eurodollar futures.

(b) Calculate the prices for a series 6 x 12, 12 x 18, 18 x 24 of FRA using rates calculated in part (a).

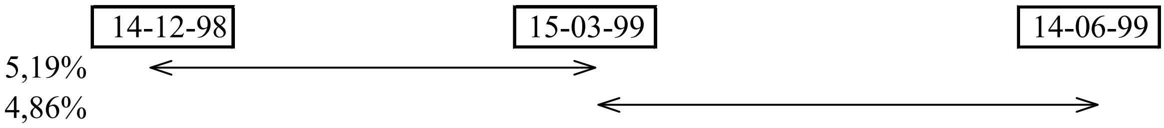
Solution

(a)

A 3 x 6 FRA rate is calculated as a weighted average of December and March Eurodollar futures rates. The number of days before and after March futures contract divided by the number of days for a FRA transaction are used as weights.



Eurodollar futures contracts



The FRA rates for a series of seven contracts:

FRA	Method of calculation
3 x 6	(5,19% * 23 days + 4,86% * 66 days) : 89 days = 4,95%
6 x 9	(4,86% * 25 days + 4,86% * 67 days) : 92 days = 4,86%
9 x 12	(4,86% * 24 days + 4,89% * 68 days) : 92 days = 4,88%
12 x 15	(4,89% * 23 days + 5,16% * 69 days) : 92 days = 5,09%
15 x 18	(5,16% * 22 days + 5,02% * 68 days) : 90 days = 5,05%
18 x 21	(5,02% * 30 days + 5,09% * 62 days) : 92 days = 5,07%
21 x 24	(5,09% * 29 days + 5,15% * 63 days) : 92 days = 5,13%

Calculations are made using the following table

Futures maturity	Futures rate	FRA 1st date	Futures maturity	FRA 2nd date	Days (p_1)	Days (p_2)	Forward rate
12-98	5,19%	20-11-98	14-12-98	20-02-99	24	68	5,25%
03-99	4,86%	20-02-99	15-03-99	20-05-99	23	66	4,95%
06-99	4,86%	20-05-99	14-06-99	20-08-99	25	67	4,86%
09-99	4,89%	20-08-99	13-09-99	20-11-99	24	68	4,88%
12-99	5,16%	20-11-99	13-12-99	20-02-00	23	69	5,09%
03-00	5,02%	20-02-00	13-03-00	20-05-00	22	68	5,05%
06-00	5,09%	20-05-00	19-06-00	20-08-00	30	62	5,07%
09-00	5,15%	20-08-00	18-09-00	20-11-00	29	63	5,13%

(b)

FRA	Method of calculation
6 x 12	(1 + 4,86% * 92/360) (1 + 4,88% * 92/360) = (1 + FRA 6x12 * 184/360) Thus FRA 6x12 = 4,90%.
12 x 18	(1 + 5,09% * 92/360) (1 + 5,05% * 90/360) = (1 + FRA 12x18 * 182/360) Thus FRA 12x18 = 5,10%.
18 x 24	(1 + 5,07% * 92/360) (1 + 5,13% * 92/360) = (1 + FRA18 x24 * 184/360) Thus FRA 18x24 = 5,13%.

Problem 2. Pricing Interest Rate Swap Off the FRA Curve

The current FRA term structure is

FRA	Rate	Days (p_t)
0 x 6	5,1331%	181
6 x 12	4,9014%	184
12 x 18	5,1036%	182
18 x 24	5,1324%	184

The notional principal of the swap is $100 million.

(a) Determine the fixed rate on the two-year interest swap using FRA rates and the following day-count conventions: "30/360", "actual/365" and "actual/360".

(b) Determine the fixed rate on the two-year interest swap under the assumption that the fixed-rate receiver will pay up-front fee equal to 2% of the notional principal ?

Solution

(a)

	K_t	$p_t=d_t-d_{t-1}$	${}_tf_{t-1}$	$\frac{p_t}{360}$	$1+{}_tf_{t-1}\frac{p_t}{360}$	k_t	a_t	$\sum_{t=1}^{T} {}_tf_{t-1}\frac{p_t}{360}K_t a_t$
0 x 6	100	181	5,13%	0,503	102,58%	102,58%	97,48%	2,5
6 x 12	100	184	4,90%	0,511	102,51%	105,15%	95,10%	4,9
12 x 18	100	182	5,10%	0,506	102,58%	107,86%	92,71%	7,3
18 x 24	100	184	5,13%	0,511	102,62%	110,69%	90,34%	9,7

The fixed swap rates for semiannual settlements are not equal to fixed swap rates for quarterly settlements.

	$\sum_{t=1}^{T}\frac{1}{2}K_t a_t$	$s_{30/360}$	$\sum_{t=1}^{T}\frac{p_t}{365}K_t a_t$	$s_{a/365}$	$\sum_{t=1}^{T}\frac{p_t}{360}K_t a_t$	$s_{a/360}$
0 x 6	48,7	5,1616%	48,3	5,2044%	49,0	5,1331%
6 x 12	96,3	5,0869%	96,3	5,0874%	97,6	5,0177%
12 x 18	142,6	5,1108%	142,5	5,1157%	144,5	5,0456%
18 x 24	187,8	5,1434%	188,1	5,1370%	190,7	5,0666%

The fixed swap rate on a two-year interest swap is 5,1434% (30/360 basis), 5,1370% (a/365 basis), and 5,0666% (a/360 basis).

(b)

As the fixed-rate receiver pays an up-front fee, the swap fixed rates are lower.

	$\sum_{t=1}^{T}\frac{1}{2}K_t a_t$	$s_{30/360}$	$\sum_{t=1}^{T}\frac{p_t}{365}K_t a_t$	$s_{a/365}$	$\sum_{t=1}^{T}\frac{p_t}{360}K_t a_t$	$s_{a/360}$
0 x 6	48,7	1,0584%	48,3	1,0672%	49,0	1,0525%
6 x 12	96,3	3,0099%	96,3	3,0102%	97,6	2,9690%
12 x 18	142,6	3,7087%	142,5	3,7123%	144,5	3,6614%
18 x 24	187,8	4,0785%	188,1	4,0734%	190,7	4,0176%

Problem 3. Pricing and Valuation of FRA

The current term structure of WIBOR is

92 -day WIBOR	5,00%
181 -day WIBOR	5,20%

The notional principal is $100 000.

(a) Calculate FRA.

(b) It is 61 days later and the relevant term structure is

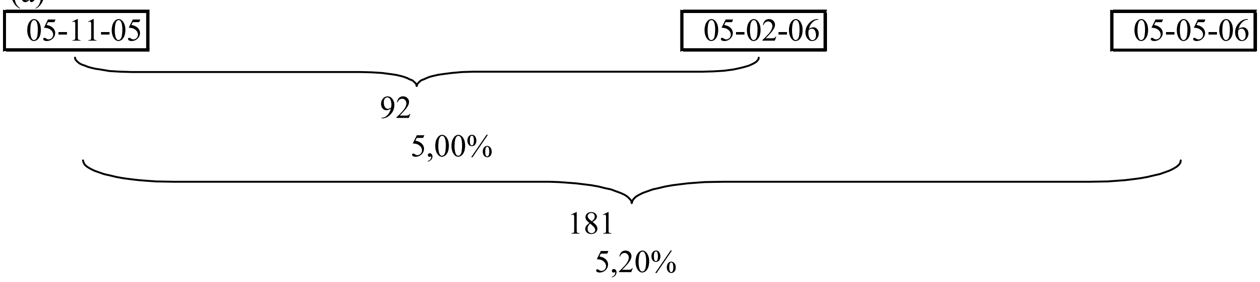
31 -day WIBOR	4,99%
120 -day WIBOR	4,84%

Determine the market value of the FRA.

(c) On the expiration day, 89-day WIBOR is 4,00%. Determine the payment.

Solution

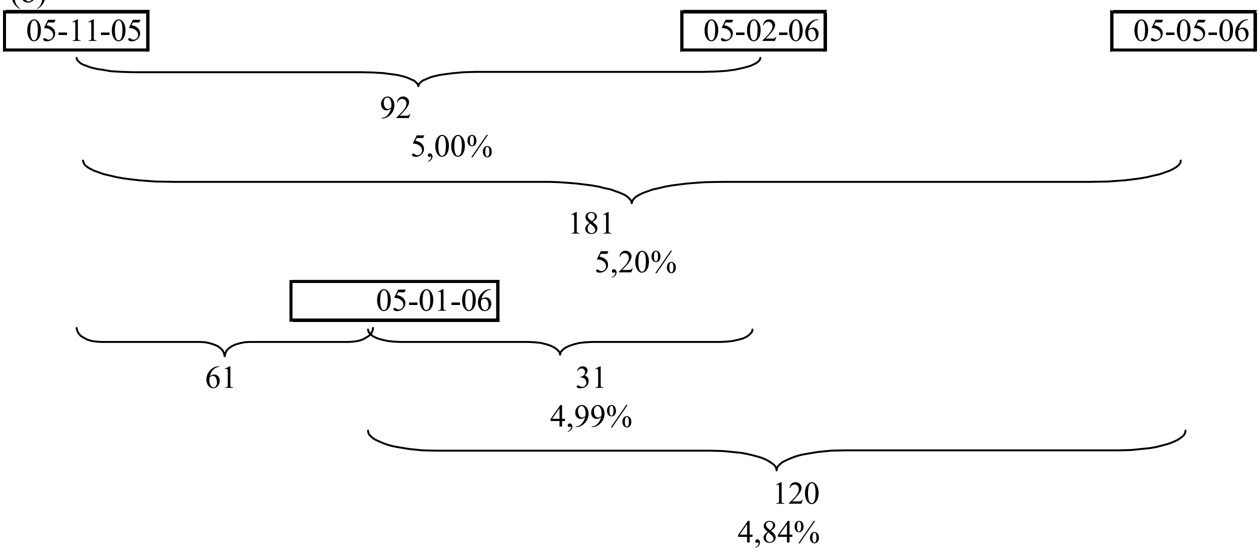
(a)



The FRA rate is:

$$ {}_2f_1 = \left[\frac{\left(1 + \frac{z_2 t_2}{365}\right)}{\left(1 + \frac{z_1 t_1}{365}\right)} - 1 \right] \frac{365}{t_2 - t_1} \qquad = \qquad 5,34\% $$

(b)



The value of FRA will be

$$V_t = \left(\frac{1}{1 + \frac{z_1' t_1'}{365}} - \frac{1 + {}_2f_1 \frac{t_2 - t_1}{365}}{1 + \frac{z_2' t_2'}{365}} \right) K \quad = \quad -137{,}722$$

or

$${}_2f_t \quad = \quad 4{,}77\%$$

$$V_t = \left(\frac{{}_2f_t - {}_2f_1}{1 + \frac{z_2' t_2'}{365}} \frac{t_2 - t_1}{365} \right) K \quad = \quad -137{,}722$$

(c)
At expiration, the payoff is

$$V_t = \left(1 - \frac{1 + {}_2f_1 \frac{t_2 - t_1}{365}}{1 + \frac{z_2'' t_2'}{365}} \right) K \quad = \quad -323{,}451$$

or

$$V_t = \left(\frac{z_2'' - {}_2f_1}{1 + \frac{z_2'' t_2'}{365}} \frac{t_2 - t_1}{365} \right) K \quad = \quad -323{,}451$$

Problem 4. Pricing and Valuation of IRS

Consider a one-year interest swap with semiannual payments.
(a) Determine the fixed rate on the swap.
The current structure of WIBOR spot rates is given as follows.

Days	184	365
WIBOR	4,50%	4,60%

(b) 153 days later, the term structure is as follows:

Days	31	212
WIBOR	4,99%	4,77%

Determine the market value of the swap from the perspective of the party paying the fixed rate and receiving the floatnig rate. Assume the notional principal of $100 000 million.

Solution

(a)

$$s_T = \frac{1 - \dfrac{1}{1 + z_T \dfrac{d_T}{365}}}{\sum_{t=1}^{T} \dfrac{\dfrac{d_t - d_{t-1}}{365}}{1 + z_t \dfrac{d_t}{365}}} \qquad = \qquad 4{,}55\%$$

d_t	184	365	
z_t	4,50%	4,60%	
$1/(1+z_t d_t/365)$	0,9778	0,9560	
$d_t - d_{t-1}$	184	181	
$(d_t - d_{t-1})/365$	0,5041	0,4959	Σ
$(d_t - d_{t-1})/365/(1+z_t dt/365)$	0,4929	0,4741	0,9670

(b)

$$MTM_t = \frac{{}_t f_{t-1} \dfrac{p_t}{365} + 1}{1 + z_t \dfrac{d_t}{365}} - \sum_{t=1}^{T} \frac{s_T \dfrac{p_t}{365}}{1 + z_t \dfrac{d_t}{365}} - \frac{1}{1 + z_T \dfrac{d_T}{365}}$$

The present value of floating payments

Number of days	184
Floating rate (least reference date)	4,50%
Cash flows	1,0227
Number of days	31
Spot rate	4,99%
Present value factor	0,996
Discounted cash flow	1,0184

The present value of remaining fixed payments:

Number of days	184	181	
Swap fixed rate	4,55%	4,55%	
Cash flows	0,0229	1,0226	
Number of days	31	212	
Spot rate	4,99%	4,77%	
Present value factor	0,9958	0,9730	Σ
Discounted cash flow	0,0228	0,9950	1,0178

Difference 0,0006 x 100 000 = 58