

## 4. Levered and Unlevered Cost of Capital. Tax Shield. Capital Structure

### Problem 26

A company is considering a totally different investment project in terms of risk.

The identified pure play company XYZ has the following characteristics:

Levered beta 1,40,

Market value of debt \$20 million

Market value of equity \$80 million

Tax rate = 20%.

Risk free interest rate is 4% and the expected return on the market is 8%.

- (a) What is the cost of equity for the pure play.  
 (b) What is the unlevered beta for the pure play.  
 (c) What is the levered project beta, if a project is financed in 50% with debt.  
 (d) What is the cost of equity for a new investment project?

### Solution

(a)

$$R_E = R_f + \beta_E (R_M - R_f) = 9,60\%$$

(b)

$$\beta_A = \frac{\beta_{ES}}{1 + (D_S/E_S)(1 - T_S)} = 1,17$$

(c)

The levered project beta is

$$\beta_{proj} = \beta_U + \beta_U (1 - T_{proj}) \frac{D_{proj}}{E_{proj}} = 2,10$$

(d)

The cost of equity for a new project is

$$R_{Eproj} = R_F + \beta_{proj} (R_M - R_F) = 12,40\%$$

**Problem 27**

ABC Corp., a furniture manufacturer, has decided to become a player in the textile industry. ABC Corp. has identified a pure-play company, Textor Inc., with the following characteristics:

$$\beta_E = 1,2$$

Debt = \$10 million of \$1,000 bonds maturing in two years with an 10,0% coupon (interest paid semiannually) and a present yield to maturity of 9,0%

Equity = 2 million common shares outstanding; yesterday's close was 35.5/8

Tax rate = 20%

If the risk-free rate is 5% and the expected return on the market is 10%, what discount rate should ABC Corp. use to value its new investment ?

**Solution****Market value of debt**

Semiannual rate of interest 5,00%

YTM semiannually 4,50%

1	2	3	4
50,000	50,000	50,000	1050,000
47,847	45,786	43,815	880,489

Cash flow discounted at 4,50%

Market price of one bond 1017,938

times number of bonds 10 000

Market value of debt 10 179 376

**Market value of equity** 71 250 000

$$\beta_A = \beta_E / [1 + (D/E)(1-T)] \quad 1,077$$

The appropriate discount rate is therefore

$$RRR = 9\% + 1,077 \times (10\% - 5\%) = 10,4\%$$

**Problem 28**

The beta of equity is 1,2 and the beta of debt is 0,5.

What must be the beta of an unlevered firm with the following debt-equity ratios?

Assume Harris and Pringle formula is used.

(a) 0,50

(b) 1,00

**Solution**

$$\beta_E E + \beta_D D = \beta_U (E + D)$$

$$\beta_U = \beta_E \frac{E}{(E + D)} + \beta_D \frac{D}{(E + D)}$$

D/E	D/A	E/A	$\beta_U$
0,5	0,33	0,67	0,97
1	0,50	0,50	0,85

$$D/A = D/E / (D/E + 1) = D/E / (D/E + E/E) = D/E / A/E$$

**Problem 29**

The Smarties Co. is in the 20% tax bracket, and its current market value is \$100 million. Assume bankruptcy is costless.

(a) What will annual tax savings from interest deductions be if it issues \$50 million of five-year bonds at a 10% interest rate and uses the proceeds to retire equity? What will be the value of the firm?

(b) What will annual tax savings from interest deductions be if it issues \$50 million of perpetual bonds at a 10% interest rate and uses the proceeds to retire equity? What will be value of the firm?

**Solution**

(a)

Annual interest	5			
Annual tax savings	1			
PV(10% ; 5; -1)	3,791			
Value	100,000	+	3,791	= 103,791
Value of equity				53,791

(b)

Annual interest	5			
Annual tax savings	1			
1,000 / 10%	10,000			
Value	100,000	+	10,000	= 110,000
Value of equity				60,000

**Problem 30**

A young investor is going to enter the optical-network business. She expects that her initial investment will be \$100,000. The tax rate is expected to be 20%.

The young investor estimates that her annual operating income will be \$20,000 for the next 5 years, and her required rate of return will be 10%. She is considering three different capital structures for her business: 100% equity, 50% equity - 50% debt, and 10% equity - 90% debt. The debt, if issued, will mature in 5 years and carry a 10% annual coupon in the second scenario and a 11% annual coupon in the third scenario.

- (a) Which capital structure will result in the greatest total income to security-holders ?  
 (b) Determine the firm's market-value balance sheet under each different capital structure.  
 (c) Which capital structure should young investor choose ? Why ?

**Solution**

- (a)
- | Capital structure    | 100 000 | 50 000 | 10 000 |
|----------------------|---------|--------|--------|
| Equity               | 100 000 | 50 000 | 10 000 |
| Debt                 | 0       | 50 000 | 90 000 |
| <hr/>                |         |        |        |
| Net income statement |         |        |        |
| NOI                  | 20 000  | 20 000 | 20 000 |
| Interest             | 0       | 5 000  | 9 900  |
| Taxable income       | 20 000  | 15 000 | 10 100 |
| Taxes                | 4 000   | 3 000  | 2 020  |
| NI                   | 16 000  | 12 000 | 8 080  |
| <hr/>                |         |        |        |
| Income to:           |         |        |        |
| Bondholders          | 0       | 5 000  | 9 900  |
| Stockholders         | 16 000  | 12 000 | 8 080  |
| Total                | 16 000  | 17 000 | 17 980 |
| ROE = NI / E         | 16,0%   | 24,0%  | 80,8%  |
| ROA = NOI (1-T) / A  | 16,0%   | 16,0%  | 16,0%  |
- The 10% E - 90%D capital structure provides the greatest total income to securityholders.
- (b)
- |   |         |         |         |
|---|---------|---------|---------|
| Tax shields   | 0       | 3 791   | 7 506   |
| =PV(coupon rate;5 years;-tax rate*coupon rate*debt) |         |         |         |
| Assets  | 100 000 | 103 791 | 107 506 |
| Equity  | 100 000 | 53 791  | 17 506  |
| Debt  | 0       | 50 000  | 90 000  |
- (c) The 10% equity / 90% debt capital structure provides the greatest total market value.  
 Note that the value of the levered firm exceeds the value of the same firm without financial leverage.  
 The difference between the two is the present value of the interest tax shield.

**Problem 31**

You are trying to determine whether or not there is an optimal capital for a new firm. You need to raise a total of \$1 000,000, and you expect the tax rate to be 20%. Any debt financing will be with perpetuities. You have estimated the present value of the costs of bankruptcy to be \$100,000, and the estimated probabilities of bankruptcy under the the following possible capital structures are:

Debt-equity (%)	0	10	20	50	80	90
Probability of bankruptcy (%)	1	5	10	40	56	96

- (a) What is the value of the firm under each of the capital structures if bankruptcy costs are ignored?  
 (b) What is the value of the firm under each of the capital structures if the expected value of bankruptcy costs are included?

**Solution**

(a)

D/E	0,0%	10,0%	20,0%	50,0%	80,0%	90,0%
D/A	0,0%	9,1%	16,7%	33,3%	44,4%	47,4%
$V_U=A$	1000,0	1000,0	1000,0	1000,0	1000,0	1000,0
$PV(TD)=D/A * T * A$	0,0	18,2	33,3	66,7	88,9	94,7
$V_L= D+E$	1000,0	1018,2	1033,3	1066,7	1088,9	1094,7

(b)

$V_U=A$	1000,0	1000,0	1000,0	1000,0	1000,0	1000,0
$PV(TD)=TD$	0,0	18,2	33,3	66,7	88,9	94,7
- PV(c)	1,0	5,0	10,0	40,0	56,0	96,0
$V_L= D+E$	999,0	1013,2	1023,3	1026,7	1032,9	998,7