

### 3. Cost of Equity. Cost of Debt. WACC

#### Problem 15

Suppose you have decided to invest in some combination of ABC Corp. and DEF Inc. ABC Corp. has an expected return of 20% and a standard deviation of returns of 5%. DEF Inc. has an expected return of 6% and a standard deviation of returns of 3%.

The correlation between the returns of Transcontinental and Axis is -0,1.

(a) Fill in the following table:

Proportions invested in ABC Corp. (%)	Expected Return on Portfolio	Standard Deviation of Returns
100%	20%	5%
80%		
60%		
40%		
20%		
0%	6%	3%

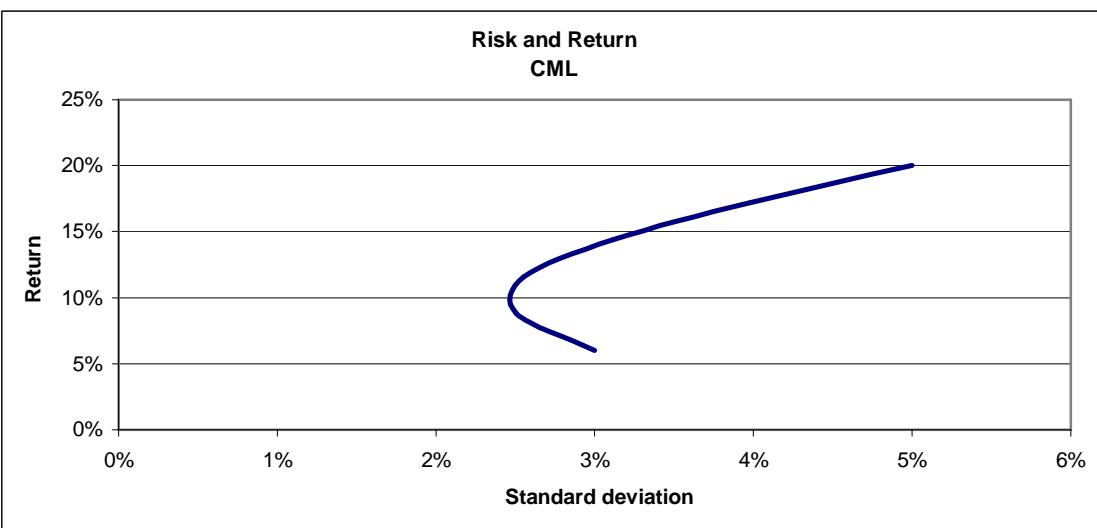
(b) Which portfolios can you exclude from consideration if you are risk averse ?

#### Solution

(a)

	Return	St.dev.
100%	20,00%	5,00%
80%	17,20%	3,98%
60%	14,40%	3,12%
40%	11,60%	2,55%
20%	8,80%	2,51%
0%	6,00%	3,00%

(b) 0,000



You will not buy 100% DEF, since the combination of 80% of DEFs and 20% of ABC has greater expected return and lower risk. You cannot exclude any other combination.

**Problem 16**

The market will have the following different returns under different economic conditions:

<i>State of the economy</i>	<i>Probability</i>	<i>Return on the market</i>
Good	0,3	0,180
Mediocre	0,5	0,144
Poor	0,2	0,090

Government T-bills will return 3% regardless of the state of the economy.

(a) Find the expected returns and the standard deviations of the market and T-bills.

(b) Complete the following table and graph results:

<i>Proportion invested in T-bills (%)</i>	<i>Proportion invested in market (%)</i>	<i>Expected Return on Portfolio</i>	<i>Standard Deviation of Portfolio</i>
100%	0%	3%	0%
75%	25%		
50%	50%		
25%	75%		
0%	100%		

**Solution**

a)

$E(M)$	14,40%	$E(T)$	3%
$D^2(M)$	0,10%	$D^2(T)$	0
$D(M)$	3,12%	$D(T)$	0

b)

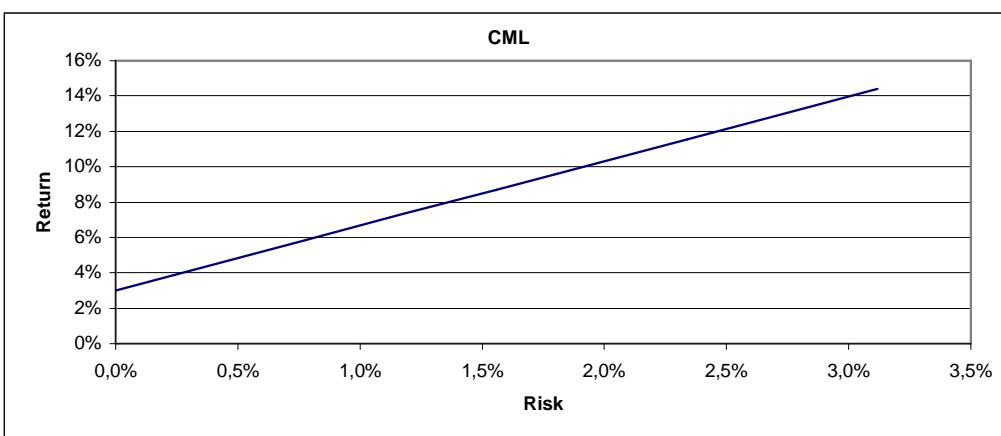
The expected return from a two-asset portfolio, is given as

$$E(r_p) = w_1 r_1 + w_2 r_2$$

The variance of a two-asset portfolio is calculated as

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$$

T-bills	Market	Exp.Ret.	Variance	Standard Deviation
100,0%	0,0%	3,00%	0,00%	0,0%
75,0%	25,0%	5,85%	0,01%	0,8%
50,0%	50,0%	8,70%	0,02%	1,6%
25,0%	75,0%	11,55%	0,05%	2,3%
0,0%	100,0%	14,40%	0,10%	3,1%



**Problem 17**

The board of directors of TYSCO, located in Hartford, is considering opening up a new branch in New Haven. They have decided to use the rates of return on the Hartford store and on the S&P 500 during the last ten years to estimate the store's beta. Accordingly, they have accumulated the following information:

Year	Risk Premium on Hartford Store (%)	Risk Premium on the Market (%)
1	8	5
2	6	2
3	3	-1
4	9	4
5	6	3
6	5	1
7	6	2
8	-8	-2
9	14	9
10	6	8

(a) Based on this information, estimate the Hartford store's beta.  
 (b) Suppose that the risk-free rate is expected to remain at 5% and the market return is estimated to be 10% for the indefinite future. What discount rate (cost of equity) should the board of directors use in deciding whether to open the new store?

**Solution**

(a)

Year	TYSCO	Deviation of returns	Squared deviations	Market	Deviation of returns	Squared deviations	Product of
1	8,000%	2,500%	0,063%	5,000%	1,900%	0,036%	0,048%
2	6,000%	0,500%	0,002%	2,000%	-1,100%	0,012%	-0,005%
3	3,000%	-2,500%	0,063%	-1,000%	-4,100%	0,168%	0,103%
4	9,000%	3,500%	0,123%	4,000%	0,900%	0,008%	0,032%
5	6,000%	0,500%	0,002%	3,000%	-0,100%	0,000%	-0,001%
6	5,000%	-0,500%	0,003%	1,000%	-2,100%	0,044%	0,011%
7	6,000%	0,500%	0,002%	2,000%	-1,100%	0,012%	-0,005%
8	-8,000%	-13,500%	1,823%	-2,000%	-5,100%	0,260%	0,689%
9	14,000%	8,500%	0,723%	9,000%	5,900%	0,348%	0,502%
10	6,000%	0,500%	0,002%	8,000%	4,900%	0,240%	0,025%
Average (sum/n)	5,500%			3,100%			
Variance & Covariance (sum/n-1)			0,312%			0,125%	0,155%
Standard deviation			5,583%			3,542%	

$$\text{Correlation coefficient } \rho_{im} = \frac{\sigma_{im}}{\sigma_i \sigma_m} = 0,155\% / (5,583\% \times 3,542\%) = 78,390\%$$

$$\text{Beta } \beta_i = \frac{\rho_{im} \sigma_i \sigma_m}{\sigma_m^2} = \frac{\sigma_{im}}{\sigma_m^2} = 78,390\% \times 5,583\% \times 3,542\% / 0,125\% = 1,236$$

$$\text{or } 0,155\% / 0,125\% = 1,236$$

$$\text{(b) } 5,000\% + 1,236 \times (10,000\% - 5,000\%) = 11,18\%$$

**Problem 18**

You are considering an investment in ABC Corp. Your statistical analysis has shown that the variance of ABC Corp. is 0,5, while that of the market is 0,5. The correlation coefficient between ABC Corp. return and that of the market is 0,80. What is ABC Corp. beta ?

**Solution**

$$\beta_j = \frac{\sigma_{jm}}{\sigma_m^2} = \rho_{jm} \frac{\sigma_j}{\sigma_m} \quad 0,800$$

**Problem 19**

ABC Corp. shares have a beta of 1,2, are currently selling for \$1 each, and pay no dividends. The risk-free rate is 4%, and the expected market return is 8%.

- (a) What do you expect ABC Corp. shares to trade for one year from today ?
- (b) If the expected market return is 10% (rather than 8%), what do you expect ABC Corp. shares to trade for one year from today ?

**Solution**

(a)	Expected return on the stock	8,8%
	Expected price	1,088
(b)	Expected return on the stock	11,2%
	Expected price	1,112

**Problem 20**

A stock has a beta of 1,5, and the risk free rate is 4%. What is the expected return on the stock in the following cases ?

- (a) The expected return on the market is 8%.
- (b) The expected return on the market is 10%.

**Solution**

Expected rate of return on asset j is calculated as

$$R_j = R_f + \beta_j [R_M - R_f]$$

The expected return

	on market	on stock
(a)	8%	10%
(b)	10%	13%

**Problem 21**

A year ago you purchased some stock that had a beta of 1,5. You know that the T-bill rate has remained at 4% throughout the year.

You hear that the market return was 8% during the last year.

(a) Given only this information, what do you expect the return on your stock to be ?

(b) If the market return was 3% instead of 8%, what would the return be?

**Solution**

Expected rate of return on asset j

$$R_j = R_f + \beta_j [R_M - R_f]$$

a) 10%  
b) 3%

**Problem 22**

The ABC Corp. has a beta of 1,20 and pays no dividends. The expected market return is 8%, and the T-bill rate is expected to remain constant at 4%. You expect ABC Corp. shares to be worth \$1,088 in one year.

Using the CAPM, find the value of a share today.

**Solution**

First compute the required rate of return, then discount the end-of-period price back to the current time:

Expected return on the stock 8,8%  
The present price 1,00

**Problem 23**

Stock A has a beta of 1,5, and stock B has a beta of 1,5. You invest 120% of your money in stock A and the rest in stock B.

(a) What is the beta of your portfolio?  
(b) If the market return is 10% and the risk-free rate is 5%, what is the expected return on your portfolio?

**Solution**

(a) Beta for a portfolio

$$\beta_p = \sum w_j \beta_j \quad 1,5$$

(b)

Expected rate of return on portfolio

$$R = R_f + \beta [R_M - R_f] \quad 12,50\%$$

**Problem 24**

The AAA stock has an expected return of 5%, the BBB has an expected return of 5,2% and CCC stock has an expected return of 5,7%.

The betas of these three firms are 0,9,1,2, and 1,4.

The T-bill rate is 3%, and the expected market return is 5%.

Which stocks are underpriced ?

**Solution**

	AAA	BBB	CCC
Expected return	5,00%	5,20%	5,70%
CAPM	4,80%	5,40%	5,80%
	underpriced	overpriced	overpriced

The undepreciated stock has a higher expected return than the market estimate.

**Problem 25**

Long term debt

Value on Balance Sheet	4 000,000
Market Value of the debt	3 600,000
Stated rate of interest	10%
Effective rate (or YTM)	12%

Shareholders equity

Common shares (on the Balance Sheet)	2 500,000
Rertained Earnings	3 500,000
Market value of shares	8 400,000
Beta of the firm's shares	1,500
Risk free rate of return	6%
Expected market return	12%

Income tax rate

40%

Calculate the weighted average cost of capital.

**Solution**

Long term debt	30,0%	7,2%	2,2%
Shareholders equity	70,0%	15,0%	<u>10,5%</u>
			12,7%