

4. Levered and Unlevered Cost of Capital. Tax Shield. Capital Structure

1.1 Levered and Unlevered Cost of Capital

Levered company and CAPM

The cost of equity is equal to the return expected by stockholders. The cost of equity can be computed using the capital asset pricing model (CAPM), the arbitrage pricing theory (APT) or some other methods.

According to the CAPM, the expected return on stock of an levered company is

$$(1) \quad R_E = R_F + \beta_E (R_M - R_F)$$

where

R_E is the expected rate of return on stock of an levered company (levered cost of equity capital),

R_F is the risk-free return,

β_E is the beta coefficient of stock of an levered company, and measures the volatility of the stock's returns relative to the market's returns (systematic risk), it is called levered beta,

R_M is the expected return on the market portfolio,

$(R_M - R_F)$ is the market risk premium for bearing one unit of market risk.

Unlevered company and CAPM

Required rate of return for the stockholders R_U of an unlevered company (a company without debt) can be also expressed as a function of risk free-rate of return and market premium risk. This time unlevered beta coefficient is used:

$$(2) \quad R_U = R_F + \beta_U (R_M - R_F)$$

where:

R_U is the expected rate of return on stock of an unlevered company (unlevered cost of capital),

β_U is the beta coefficient for an unlevered company (unlevered equity beta, all-equity beta).

1.2 Project Beta

Managers often assume that the cash flows from the project under consideration will be as risky as the cash flows from the existing operations of the firm (scale-enhancing projects). This approach is strictly valid for scale-enhancing projects. When a firm takes on a project with cash flows that do not have the same characteristics as the firm's existing assets, it should use the appropriate cost of equity for specific projects and not cost of equity for the whole company. The RRR for an investment should be project specific and not company specific. The opportunity cost for a particular project depends on the project's risk and should be determined by the market.

A project's relevant risk (β_{proj}) should be measured in terms of the relationship between the returns generated by the project and the returns on the market portfolio. Thus a project beta measures the sensitivity of changes in a project's returns to changes in the market return.

A project's beta may be estimated using historical information when it is available. When it is not available the following procedure may be used:

1. Find a publicly traded company whose business is as similar as possible to your project. The company thus identified is called a "pure-play company". It is better to work with industry betas.
2. Determine the equity beta, β_E , for the pure-play firm's stock.
3. Calculate the pure play's unlevered equity beta (asset beta), β_U , from its β_E , by explicitly adjusting the β_E for the pure play's financial leverage.
4. Calculate the project's beta β_{proj} from its unlevered equity beta β_U , by explicitly adjusting the β_U for the project's financial leverage.
5. Find the RRR for your project, using the project's beta β_{proj} from Step 4.

Using Damodaran's relationship between levered beta and unlevered beta the the following formulas are used:

$$(1) \quad \beta_U = \frac{\beta_E}{1 + \left(\frac{D_{comp}}{E_{comp}} \right) (1 - T_{comp})} \quad (\text{Step 3})$$

$$(2) \quad \beta_{proj} = \beta_U + \beta_U (1 - T_{proj}) \frac{D_{proj}}{E_{proj}} \quad (\text{Step 4})$$

where

D_{comp} and E_{comp} represent the market values of the pure-play company's debt and equity, and T_{comp} is the pure play's tax rate.

D_{proj} and E_{proj} represent the market values of the company's debt and equity used to finance the project, and T_{proj} is its tax rate.

The equity risk increases as the debt - equity ratio increases.

According to CAPM, the project's RRR with a systematic risk level proxied by β_{proj} (Step 5) is

$$(3) \quad R_{Eproj} = R_F + \beta_{proj} (R_M - R_F)$$

1.3 Capital Structure and Firm Value

The proportion of each component of capital used by a firm determines the firm's capital structure. The company's capital structure is often measured by debt-equity ratio, also called leverage ratio. A company that has no debt is called an unlevered firm; a company that has debt in its capital structure is a levered firm. How levered should the firm be? Is it possible to determine optimal capital structure?

Optimal capital structure is the debt-equity ratio, that maximizes the firm's value. Theoretically it is easy to establish the optimal structure. In practice this problem is difficult to solve.

1.3.1 Optimal Capital Structure Without Taxes

Modigliani and Miller (M&M) hypothesis 1. With riskless debt and the absence of taxes, capital structure is irrelevant. The method of financing has no impact on firm's value. As long as return on assets is not affected, the firm's value is independent of its capital structure. The most important assumptions for this irrelevance result is that capital markets have no imperfections such as taxes, brokerage fees, and bankruptcy costs.

1.3.2 Optimal Capital Structure With Taxes

Modigliani and Miller (M&M) hypothesis 2. When a company pays corporate tax, it should be totally debt financed. This is because debt provides valuable tax shields (tax savings).

1.3.3 Tax Shield

One-period tax shield (tax savings for a levered company) is equal to:

$$(3) \quad \text{Tax shield} = T * I$$

where

T is income tax rate

I is the interest payments ($I = R_F D$)

D is the market value of long-term debt

According to M&M the present value of future tax savings can be found with a simple formula:

$$(4) \quad E_S = \sum_{t=1}^n \frac{R_F D T}{(1 + R_F)^t}$$

It is assumed that interest rate is equal to risk free rate and that appropriate discount rate is the required rate of return on debt is also equal to risk free rate. It is assumed further a constant level of debt (D) and a series of equal tax savings to infinity. According to M&M theory value of tax shields is equal to:

$$(5) \quad E_S = \frac{R_F D T}{R_F} = TD$$

The value of levered firm is greater than the value of an unlevered by the present value of tax shields:

$$(6) \quad D + E = A + E_S$$

1.3.4 Financial Distress

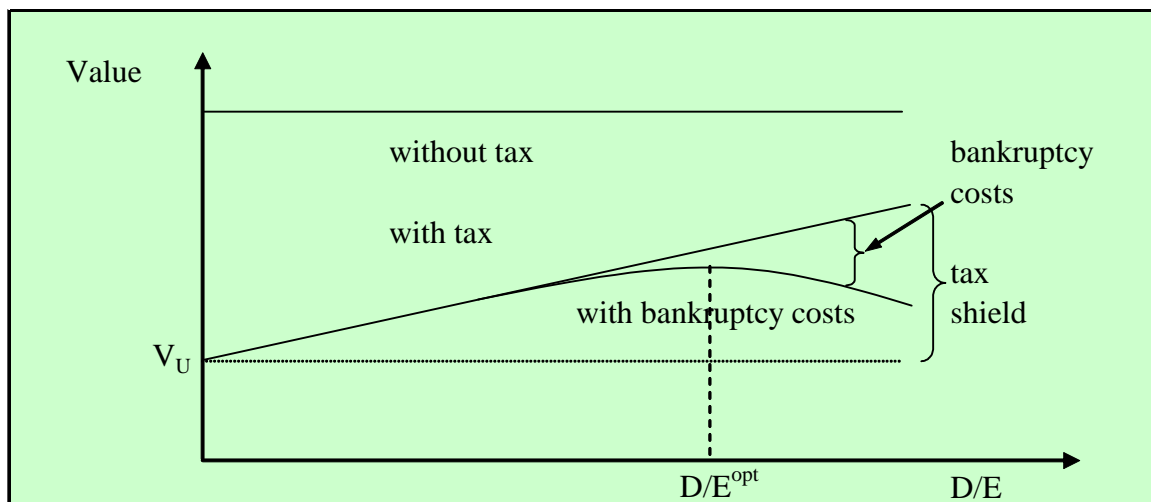
Financial Distress and Default Considerations

As the debt-equity ratio increases, the probability that a firm will be unable to meet its financial obligations also increases. If the credit risk is high, financial distress or default may happen.

The costs associated with default are often referred to as bankruptcy costs. There are two types of bankruptcy costs: direct and indirect. Direct costs are legal and accounting fees, reorganization costs, and other administrative expenses. Indirect costs are more difficult to measure. Examples of indirect are lost sales, lost profits, higher interest rates (cost of debt), and in the end the inability to invest in profitable projects because external financing sources become unavailable.

In the case of possible default, a levered firm's value is lower by the present value of expected bankruptcy costs.

$$\begin{matrix} \text{Value} \\ \text{of} \\ \text{levered firm} \end{matrix} = \begin{matrix} \text{Value} \\ \text{of} \\ \text{unlevered firm} \end{matrix} + \begin{matrix} \text{Present value} \\ \text{of} \\ \text{tax shields} \end{matrix} - \begin{matrix} \text{Present value} \\ \text{of} \\ \text{bankruptcy costs} \end{matrix}$$



1.3.5 Agency Costs

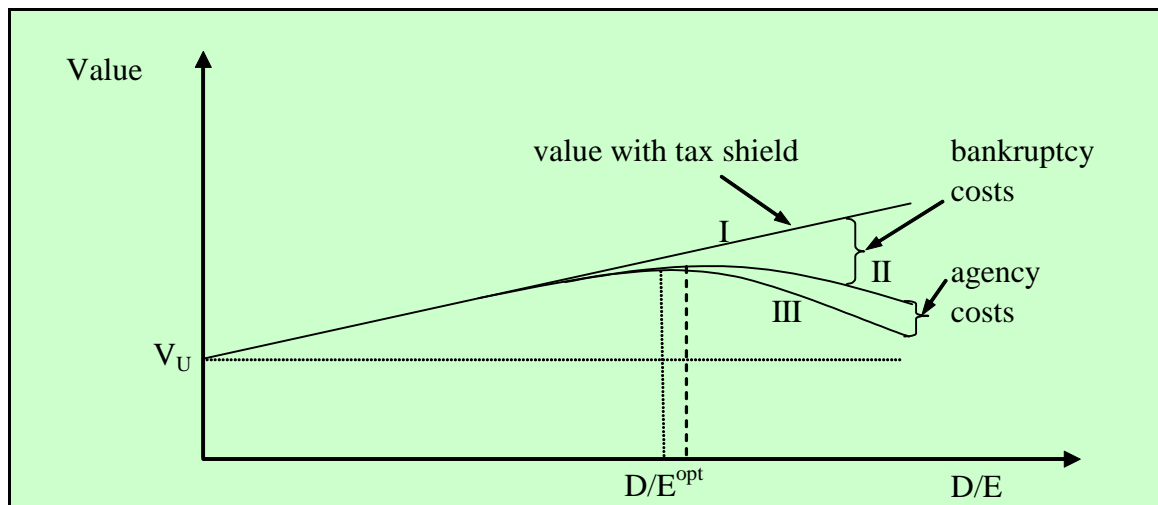
Agency Costs Considerations

Managers are expected to act in the best interest of owners, but sometimes managers act in their own interest. The conflict between these interests is called **agency costs**. Agency costs arise from the conflict between the interests of the managers and the owners of a company.

Agency costs in a levered company

It is supposed that as a firm increases its debt-equity ratio, its agency costs rise and, as a result, the value of a levered company is lower. But other viewpoints suggest that agency

costs can decrease with debt. Since it is difficult to measure agency costs, it is uncertain to what extent they influence the market value of a levered company.



1.4 Levered Beta and Tax Shield

Selection the relation between levered beta β_E and unlevered beta β_U or alternatively the appropriate way of calculating the value of tax shield E_S is essential. It is a mistake for example to use Damodaran's formula for levered beta and in the same time Miller and Modigliani formula for a tax shield.

The precise relationship between levered beta β_E and unlevered beta β_U determines cost of equity capital R_E and in the same time uniquely determines the value of tax shield E_S . The chosen relationship or interdependent particular tax shield definition influences the value of an asset or NPV in capital budgeting. Different tax shield definitions are presented by:

- 1) Modigliani i Miller¹,
- 2) Myers²,
- 3) Miller³,
- 4) Miles and Ezzel⁴,
- 5) Harris and Pringle⁵,
- 6) Damodaran⁶,
- 7) Fernandez - practitioners method⁷.

¹ F. Modigliani, M. Miller (1963), *Corporate Income Taxes and the Cost of Capital: A Correction*, American Economic Review, June 1963, s. 433-443.

² S.C. Myers, S.C., "Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting", *Journal of Finance*, March 1974, s. 1-25

³ M. Miller, "Debt and Taxes", *Journal of Finance*, May 1977, s. 261-276.

⁴ J.A. Miles, J.R. Ezzell, "The Weighted Average Cost of Capital", *Perfect Capital Markets and Project Life: A Clarification*, *Journal of Financial and Quantitative Analysis*, September 1980, 719-730. and Miles, J.A. and J.R. Ezzell, "Reequationing Tax Shield Valuation: A Note", *Journal of Finance* Vol XL, 5, December 1985, s. 1485-1492.

⁵ R.S. Harris i J.J. Pringle, "Risk-Adjusted Discount Rates Extensions form the Average-Risk Case", *Journal of Financial Research*, 1985, s. 237-244.

⁶ A. Damodaran, A (1994), *Damodaran on Valuation*, John Wiley and Sons, 1994, New York.

⁷ P.Fernandez, *Valuing Companies by Cashflow Discounting: Ten Methods and Nine Theories*, February 2002

- 8) Fernandez - no-cost-of-leverage,
- 9) Fernandez - with-cost-of-leverage,
- 10) Marciniak - decomposition method⁸.

⁸ Z.Marciniak, Value and Risk Management using Derivatives, Oficyna SGH. Warszawa, December 2001.

The relationship between levered beta β_E and unlevered beta β_U and in the same time uniquely determined tax shield E_S are shown in the following table

Table 1. Levered Beta and Tax Shield

No	Author	Levered Beta	Tax Shield
1.	M&M	$\beta_E = \beta_U + \left(\beta_U - \beta_D + \frac{R_D T - R_F T \frac{R_U - g}{R_F - g}}{R_M - R_F} \right) \frac{D}{E}$	$E_S = \sum_{t=1}^n \frac{R_F D T}{(1 + R_F)^t}$
2.	Myers	$\beta_E = \beta_U + (\beta_U - \beta_D) \frac{D - E_S}{E}$	$E_S = \sum_{t=1}^n \frac{R_D D T}{(1 + R_D)^t}$
3.	Miller	$\beta_E = \beta_U + \left(\beta_U - \beta_D + \frac{R_D T}{R_M - R_F} \right) \frac{D}{E}$	$E_S = 0$
4.	Miles and Ezzel	$\beta_E = \beta_U + (\beta_U - \beta_D) \left(1 - \frac{R_D T}{1 + R_D} \right) \frac{D}{E}$	$E_S = \frac{1 + R_U}{1 + R_D} \sum_{t=1}^n \frac{R_D D T}{(1 + R_U)^t}$
5.	Harris and Pringle	$\beta_E = \beta_U + (\beta_U - \beta_D) \frac{D}{E}$	$E_S = \sum_{t=1}^n \frac{R_D D T}{(1 + R_U)^t}$
6.	Damodaran	$\beta_E = \beta_U + \beta_U (1 - T) \frac{D}{E}$	$E_S = \sum_{t=1}^n \frac{R_U D T - (R_D - R_F) D (1 - T)}{(1 + R_U)^t}$
7.	Fernandez - practitioners method	$\beta_E = \beta_U + \beta_U \frac{D}{E}$	$E_S = \sum_{t=1}^n \frac{R_D D T - (R_D - R_F) D}{(1 + R_U)^t}$
8.	Fernandez - with-cost-of-leverage	$\beta_E = \beta_U + [\beta_U (1 - T) + \beta_D T] \frac{D}{E}$	$E_S = \sum_{t=1}^n \frac{(R_U T + R_F - R_D) D}{(1 + R_U)^t}$
9.	Fernandez - no-cost-of-leverage	$\beta_E = \beta_U + (\beta_U - \beta_D) (1 - T) \frac{D}{E}$	$E_S = \sum_{t=1}^n \frac{R_U D T}{(1 + R_U)^t}$
10.	Marciniak	$\beta_E = \beta_U + \left(\beta_U - \beta_D + \frac{R_D T - R_C T \frac{R_U - g}{R_E - g} \frac{B_{t-1}}{D_{t-1}}}{R_M - R_F} \right) \frac{D}{E}$	$E_S = \sum_{t=1}^n \frac{R_C B T}{(1 + R_E)^t}$

where

β_U is unlevered beta

β_E is levered beta

β_D is debt beta

R_U is unlevered cost of capital

R_E is levered cost of equity

R_F is risk-free return

R_C is coupon rate

R_D is interest rate (yield),

R_M is market rate of return

$(R_M - R_F)$ is market risk premium,

T is income tax rate

E is equity including tax shield

D is long-term debt

B is book value of debt

E_S is tax shield

g is growth of tax shield.