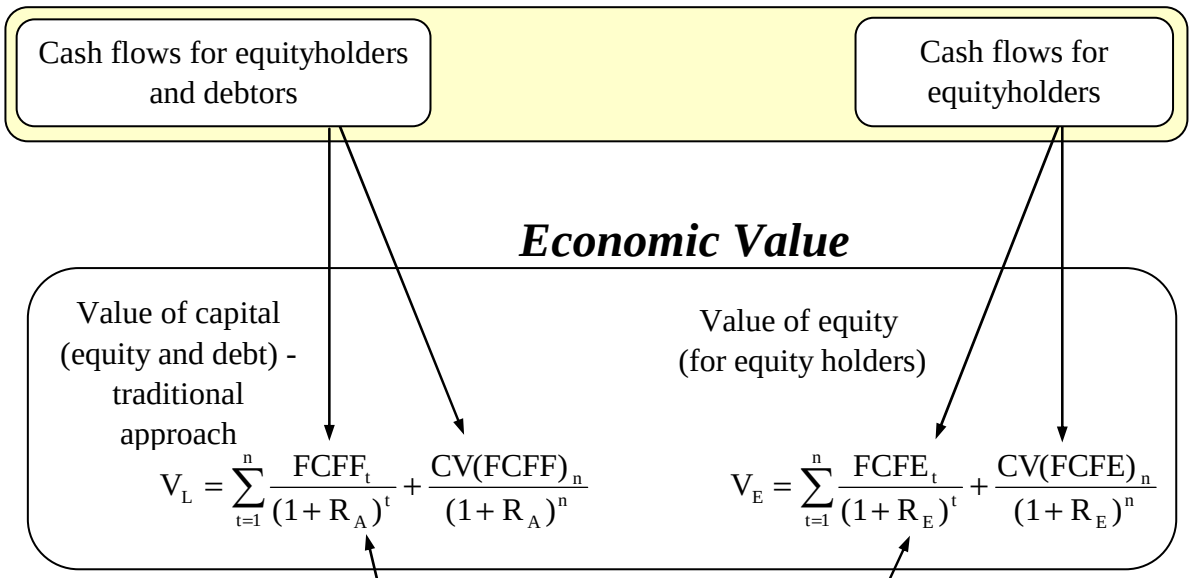
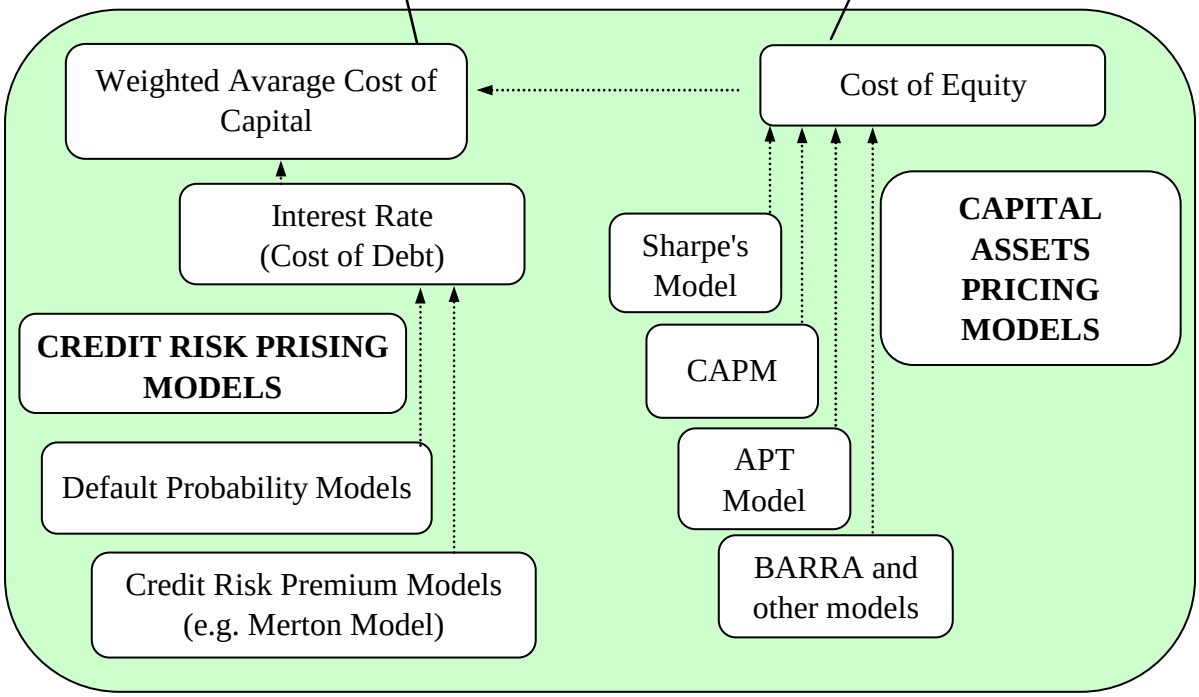
3. Cost of equity. Cost of Debt. WACC.

Cash flows Forecasts



Required Rate of Return



1.1 Cost of equity

The cost of equity is equal to the return expected by stockholders. The cost of equity can be computed using the capital asset pricing model (CAPM) or the arbitrage pricing theory (APT) model.

1.1.1 Basic Return and Risk Concepts

Return

Simple rate of return

The **relative return**, or **percent return**, for the same period (HPY, holding period yield) is

(1) $$r_t = \frac{p_t - p_{t-1} + d_t}{p_{t-1}} = \frac{p_t + d_t}{p_{t-1}} - 1 = \frac{\Delta p_{t-1} + d_t}{p_{t-1}}$$

Two different types of returns must be distinguished. An **ex ante** return is the uncertain return that an investor **expects** to get from an investment. The **ex post** or **realized** return is the certain return that an investor actually **obtains** from an investment. Investors make decisions on the benefits they expect from an investment. The actual outcomes may not match their expectations.

The equivalent annualized rate is equal to

(1) $$i = (1 + r_t)^{\frac{365}{t}} - 1$$

The log price change (continuously compounded return) of a security is defined to be the natural logarithm of its gross return:

(2) $$r_t^* = \ln(1 + r_t) = \ln\left(\frac{p_t + d_t}{p_{t-1}}\right) = \ln(p_t + d_t) - \ln(p_{t-1})$$

The **expected return from an investment** is the average return from the investment and may be calculated as the probability-weighted sum of all possible returns:

(2) $$E(r_I) = \sum_{i=1}^{k} p_i r_i$$

In this definition p_j is the probability of a particular return. Equation requires that each return be multiplied by its probability of occurrence and then all these products be added together.

The **expected return on a portfolio** is

(3) $$E(r_P) = \frac{P_1 - P_0}{P_0} = w_1 r_1 + w_2 r_2 + \cdots + w_n r_n$$

(4) $$E(r_p) = \sum_{j=1}^{n} w_j r_j$$

where w_j is the proportion (weight) invested in security j, and r_j is the expected return on security j.

The **expected return from a two-asset portfolio**, is given as

(5) $$E(r_P) = w_1 r_1 + w_2 r_2$$

Risk

The most popular and traditional measure of risk is the **variance** or the **standard deviation** of a distribution.

The variance (σ^2) of returns from an investment is the sum of the probability-weighted squared deviations of returns from the mean:

$$\sigma_I^2 = \sum_{i=1}^{k} p_i \left[r_i - E(r_I) \right]^2 \tag{3}$$

Variance is calculated by finding the expected return, finding the difference between each possible return and the expected return, squaring this value, multiplying it by the probability of that occurrence, and summing this resulting value over all possible occurrences.

The variance of a two asset portfolio is calculated as

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 \tag{6}$$

The variance of an n asset portfolio is calculated as

$$\sigma_P^2 = \mathbf{w}^T\mathbf{V}\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22}^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn}^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{7}$$

or

$$\sigma_P^2 = \mathbf{u}^T\mathbf{R}\mathbf{u} = \begin{bmatrix} w_1\sigma_1 & w_2\sigma_2 & \dots & w_n\sigma_n \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} w_1\sigma_1 \\ w_2\sigma_2 \\ \vdots \\ w_n\sigma_n \end{bmatrix} \tag{8}$$

The **standard deviation** (σ) of returns is the square root of the variance of the distribution.

$$\sigma = \sqrt{\sigma^2} \tag{4}$$

Probability Distributions

A **probability distribution** is a collection of the different possible outcomes from an uncertain variable together with the probability of each possible outcome. Probability distributions of returns are estimated using actual historical data.

The information contained in the distribution may be summarized in two simple measures:

1. expected return from an investment,
2. riskiness (or variability) of these returns.

Diversification and Portfolio Risk

Diversification is the process of reducing risks by forming a portfolio of **imperfectly correlated** securities. Investors can lower their risks by forming portfolios to get the benefits of diversification.

Nature of the Risk Components

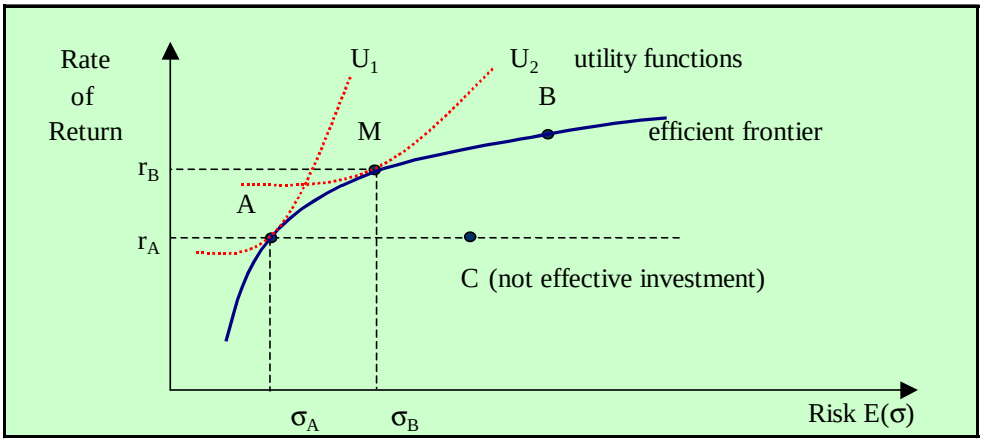
It is convenient to divide total risk (variance risk) into two distinct components: **undiversifiable risks** (the covariance risk) and **diversifiable risk** (the remaining risk in the portfolio). Undiversifiable risks are **market risks**, also known as **systematic risks** (beta risks). Market risks represent that component of total risk that are systematically dependent on the vagaries of the U.S. economy. **Unique** (diversifiable) risks are risks that are specific to a company. These are risks that can be diversified away by forming a large portfolio.

1.1.2 CAPM

Markowitz portfolio theory

The basic Markowitz model shows the relation between expected return and expected risk measured as the standard deviation of expected rate of return. The last depends on correlation between returns on assets. A investment is considered to be efficient if not other investment offers higher expected return with the same (or lower) level of risk, or lower risk with the same (or higher) expected return. The efficient frontier represents the set of portfolios that has maximum rate of return for every given level of risk, or the minimum risk for every level of return.

The utility function determines which particular portfolio on the efficient frontier gives the highest value for an investor. The optimal portfolio is the efficient portfolio that has the highest utility. There are different utility functions for conservative and aggressive investors.



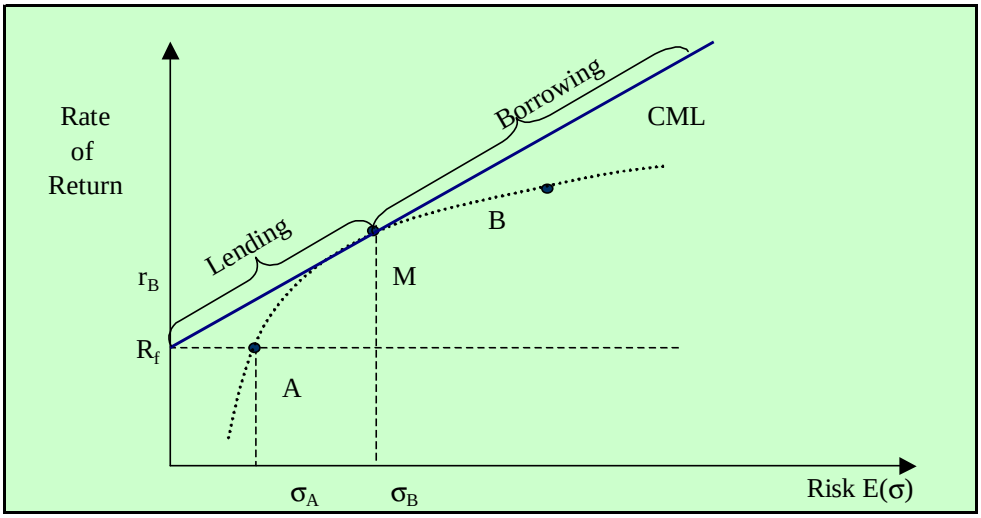
Capital Market Line

The major factor that allowed portfolio theory to develop into capital market theory is the concept of risk-free asset. It is an asset with zero variance and zero correlation with other assets. Risk-free asset provides the risk-free rate of return (R_f). The standard deviation of a portfolio that combines the risk-free asset with risky assets is the linear function of the standard deviation of the risky assets portfolio.

An investor may want to achieve higher return than is available at point M accepting higher risk. She may borrow at the risk free rate and invest in the risky asset portfolio M. Both return and risk increase linearly beyond point M. An investor may also want to achieve lower return and lower risk. This time she should invest part of her money in the risk-free asset.

The portfolio (M) that includes all risky assets is referred to as the market portfolio. It is a completely diversified. Only systematic risk remains in the market portfolio.

CML becomes the efficient frontier of portfolios. Investors may select any point on the CML. Tobin called this division of the investment decision from the financing decision the **separation theorem**. Investor initially decides to invest in the market portfolio M (*investment decision*). Then based on her risk preferences, she has to lend or borrow (*financing decision*) to attain the preferred point on the CML.



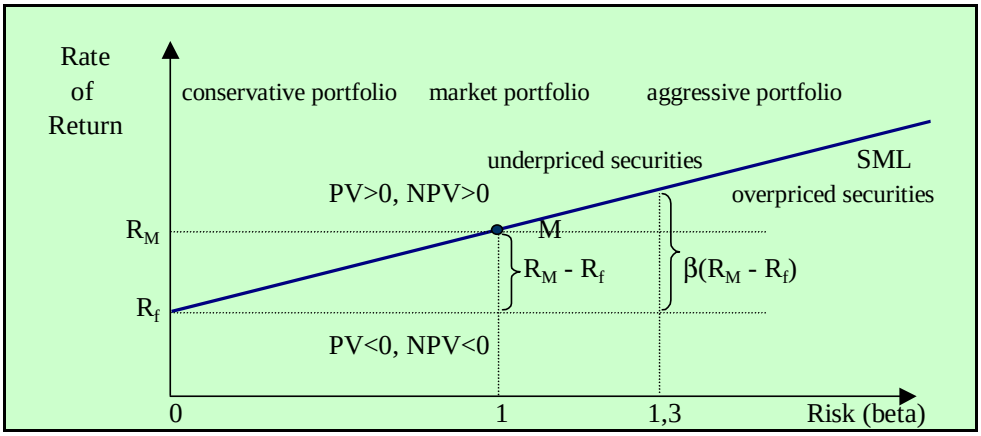
CAPM and SML

Pricing model delivers discount rate used in valuations, especially in pricing shares. CAPM delivers discount rate (RRR, required rate of return for equity holders, which is equal to the cost of equity for a company).

Because all investors want to be on the CML, an asset's covariance with the market portfolio appeared to be the relevant measure of risk. This step takes us into the **capital asset pricing model** (CAPM) and the security market line (SML). Beta is a standardized measure of systematic risk (covariance with the market portfolio is divided by the variance of the market portfolio):

$$(9) \qquad \beta_j = \frac{\sigma_{jm}}{\sigma_m^2} = \rho_{jm}\frac{\sigma_j}{\sigma_m}$$

The Security Market Line represents the relation between rate of return and risk measured by the beta coefficient. The Security Market Line reflects the risk-return combinations available for all risky assets in the capital market at a given time. Investors choose investments that are consistent with their risk preferences; some prefer only low-risk investments and others select high-risk investments.



Expected Rate of Return and Risk (CAPM)

The expected (required rate of return for shareholders, RRR, equal to the cost of equity for a company) is

(10) $$R_j = R_f + \beta_j [R_M - R_f]$$

β_j is the beta coefficient of stock i, and measures the volatility of the stock's returns relative to the market's returns
R_m is the expected return on the market portfolio (S&P 500, WIG)
R_f is the risk -free interest rate.

The term $[R_M - R_f]$ in the CAPM is called the market risk premium for bearing one unit of market risk.

Capital asset pricing model (CAPM) indicates what should be expected or required rates of return (RRR) on the risky assets (which is equal to cost of equity, discounting rate used to value equity, and investment projects).

It also shows how to create aggressive and conservative portfolios. It answers the question, which assets should be selected to achieve positive economic profits (value added, wealth created, goodwill, NPV). It is used to determine whether the asset is undervalued, properly valued or overvalued.

We can now see why CAPM is called a "pricing model". Given the expected cash flows from an investment (FCFE), the value of the asset is the present value of these expected cash flows, calculated at the discounting rate provided by the CAPM (cost of equity). An asset is priced "fairly" if the market price is equal to the equilibrium price provided by CAPM. Whenever a stock is *overvalued*, it falls below the security market line (SML); whenever it is *undervalued*, it falls above the SML.

1.1.3 Arbitrage Pricing Theory

The arbitrage pricing theory assumes that the returns on a security are affected by several factors that affect the economy. The sensitivity of stock i's return to a factor j, is the **factor beta, β_{ij}**. In a non-arbitrage economy, the following relationship should hold:

(11) $$R_j = R_f + \beta_{F1}[R_{F1} - R_f] + \beta_{F2}[R_{F2} - R_f] + ... + \beta_{Fn}[R_{Fn} - R_f]$$

In the APT framework, an asset can have as many betas as there are relevant factors. The total risk premium is the sum of risk premiums associated with each factor. It appears that the APT is a simple extension of the one-factor CAPM, in which the market portfolio is the only relevant factor.

1.2 Cost of Debt

Cost of debt is not a coupon rate or the stated rate in the loan agreement between a company and the bank. When a company issues bonds, the financial intermediary (investment bank) charges fees, called **flotation costs**, at the time the bonds are issued. To find the net proceeds to a company from issuing bonds, we subtract the flotation costs from the price of the bonds. When a company takes a loan, the bank charges provisions. The net proceeds to a company are equal to the nominal loan value less provisions.

To find the RRR for the bondholders or banks and in the same time the cost of debt for a company it is necessary to find the internal rate of return on future cash flows (interest and repayments) and the actual net proceeds to a company (with negative sign).

There is always some risk that the firm will default. It is possible to use one of two approaches:

- adjust cash flows with probabilities of default and discount them with risk free rates of return, or
- discount nominal cash flows with interest rates which are equal to risk free rates adjusted for credit risk premiums that are dependent on the rating of the company (ratings may be assumed to change - upgrades and downgrades).

It is a mistake to adjust cash flows and in the same time use interest rates that explicitly involve credit risk.

The described procedures give us the pretax required rate of return for bondholders or banks (cost of debt for a company).

In traditional approach we may have one interest rate (cost of debt), but it is also possible to derive the term structure of cost of debt for a company. To derive the term structure of cost of debt we may use the risk-free rate term structure adjusted for credit risk premium. The last may be dependent on the general economic situation forecasts and be dependent on recovery rates and also subjective assumptions on upgrades or downgrades.

Since interest payments on corporate debt are tax deductible, **debt provides a tax shield**. All we need to do to find after-tax cost of debt is **multiple the pretax cost by 1 minus the tax rate**.

1.3 WACC

The weighted average cost of capital is a discount rate used in traditional (FCFF) methods in capital budgeting projects. Perhaps the most basic intuition underlying investment decisions is that the return on investment (net assets) must exceed the cost of capital WACC (stockholders and bondholders capital) used to finance the investment.

The WACC is computed in two simple steps:

1. Compute the costs of individual sources of capital - that is, equity and all sources of debt.
2. Compute the weighted average of these individual costs, with the weights depending on the proportion of each component in the firm's capital structure. The best approach is to use market values in determining the capital structure.

$$WACC = u_1 k_1 + u_2 k_2 + ... + u_n k_n$$

u_j – weight of financial source j,
k_j – cost of financial source j (required rate of return for investors providing capital).

We have shown that it is possible to calculate term structure for cost of equity and cost of debt. So the result is obvious, we can also easily calculate the varying weighted average cost of capital for all future time steps. This leads us to non-arbitrage approach in calculating WACC.

Questions:

1. Analyze the cost of equity using the CAPM.
2. Describe the APT theory.
3. Analyze cost of debt using YTM approach.
4. Explain the treatment of floatation costs or provisions in calculating cost of debt.
5. Describe the role of taxes in cost of capital.
6. Describe the methods of calculating the weights in WACC.