2. The Time Value of Money

Problem 4

Suppose you deposit $100 in the bank today and it earns interest at a rate of 10% compounded annually. How much will be in the account 50 years from today?

Solution

In this case, $100 invested at 10-percent for 50 years accumulates to

$FV = PV(1+i)^n$ = 11739,1

Problem 5

How much money must you deposit in a savings account today to have $10,000 10 years from today if the interest rate is 5% compounded annually ?

Solution

$$PV = FV\frac{1}{(1+i)^n} = 6{,}139$$

Problem 6

A stock has paid dividends regularly for the last 10 years, starting with a $1 dividend in 1997 and increasing to $8 in 2007. If these dividends have been growing at a constant rate, what has that rate been for the last 10 years?

Solution

The exact answer is obtained by solving for i in the expression $(1+i)^{10}=8$; the equation holds when $i = 8^{0.1}-1 = 23{,}11\%$

$$i = \sqrt[n]{\frac{FV}{PV}} - 1 = 23{,}11\%$$

Problem 7

Your sister borrows $1 and promises to repay $1,5. If you want at least a 6% return on your loan, within how many years must she pay you back?

Solution

The exact answer is obtained by solving for "n" in the expression $(1{,}06)^n=1{,}5$; Taking the natural log of both sides and rearranging,

$$n = \frac{\ln\frac{FV}{PV}}{\ln(1+i)} = 6{,}96 \text{ years}$$

n = (ln 1.5 / ln 1.06) =6,96 years.

Problem 8

Compare the future value in one year of $100,000 to be invested today at 10% under the following scenarios.
(a) Annual compounding
(b) Quarterly compounding
(c) Continuous compounding

Solution

(a)	100,000*(1+10%)^1=	110,000
(b)	100,000*(1+2.5%)^4=	110,381
(c)	100,000*e^10%=	110,517

Problem 9

An annuity due (or rent annuity) is an annuity whose payments are made at the beginning of each period. What is the present value of a 20-year annuity due with $1000 payments if the discount rate is 5%?

Solution

PVA = 1000+1000 ($PVFA_{.05,19}$)

1000 + 12085 = 13085.

Twenty payments of $1000 made at the beginning of each year is the same as $1000 received now and an ordinary annuity of $1,000 received at the end of years 1,2,...,19.

Problem 10

Which of the following perpetuities represents the largest present value?

Perpetuity	*Annual amount ($)*	*Discount rate (%)*
A	100,000	5,0%
B	120,000	6,0%
C	160,000	8,0%
D	200,000	10,0%

Solution

A	B	C	D
2 000,000	2 000,000	2 000,000	2 000,000

Problem 11

Calculate the present value of a growing annuity that starts off with a payment of $100 and the subsequent annual payments grow at the annual rate of 8%. The discount rate is 10%, and the security makes a total of 5 payments.

Solution

Value of annuity

$$PV = \frac{PMT_1}{i-g}\left[1-\left(\frac{1+g}{1+i}\right)^n\right] \quad = \quad 438{,}31$$

Problem 12

A bond pays $50 interest every year plus $1000 when it matures in 12 years. You can buy the bond today for $950. What is the internal rate of return of this investment ?

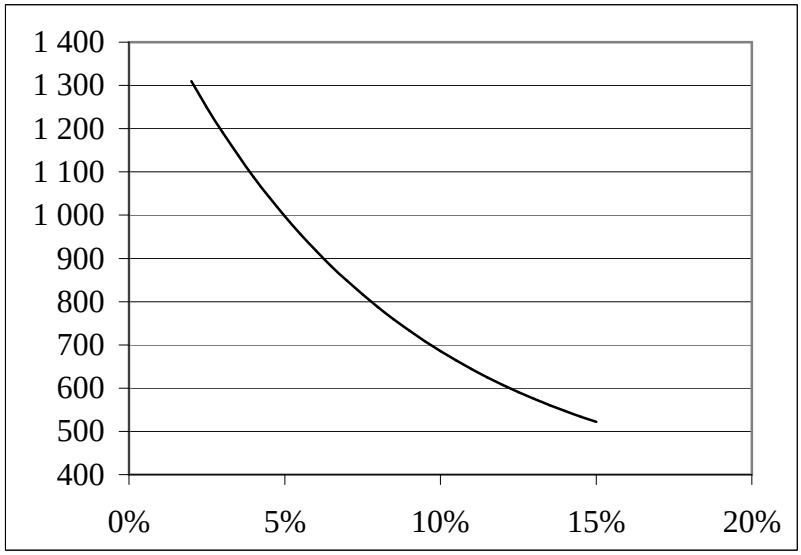
Solution

The YTM (internal rate of return) discounts the cash flows to the current market price.

0	1	2	3	4	5	6	7	8	9	10	11	12
-950	50	50	50	50	50	50	50	50	50	50	50	1 050

IRR = 5.58%

Rate	Price
2.0%	1 310
2.5%	1 249
3.0%	1 192
3.5%	1 138
4.0%	1 088
4.5%	1 041
5.0%	998
5.5%	957
6.0%	918
6.5%	882
7.0%	848
7.5%	817
8.0%	787
8.5%	759
9.0%	733
9.5%	709
10.0%	686
10.5%	664
11.0%	644
11.5%	625
12.0%	607
12.5%	591
13.0%	575
13.5%	561
14.0%	547
14.5%	534
15.0%	522



Problem 13

Loan amount $	100,000	
Interest rate	12%	per year
Period	1	year, monthly equal payments
Provision	2%	
(a) Show the loan amortization schedule.		
(b) Calculate the effective interest rate with and without a provision $2,000.		

Solution

(a)

$$PV = \frac{PMT}{(1+i)^1} + \frac{PMT}{(1+i)^2} + \ldots + \frac{PMT}{(1+i)^n} = PMT\frac{1-\left[\frac{1}{(1+i)^n}\right]}{i} = PMT\frac{(1+i)^n - 1}{i(1+i)^n}$$

Period	Payment	Interest	Principal paid off	Remaining principal
0				100,000
1	8,885	1,000	7,885	92,115
2	8,885	0,921	7,964	84,151
3	8,885	0,842	8,043	76,108
4	8,885	0,761	8,124	67,984
5	8,885	0,680	8,205	59,779
6	8,885	0,598	8,287	51,492
7	8,885	0,515	8,370	43,122
8	8,885	0,431	8,454	34,668
9	8,885	0,347	8,538	26,130
10	8,885	0,261	8,624	17,506
11	8,885	0,175	8,710	8,796
12	8,885	0,088	8,797	0

(b)

0	-100,000	-98,000
1	8,885	8,885
2	8,885	8,885
3	8,885	8,885
4	8,885	8,885
5	8,885	8,885
6	8,885	8,885
7	8,885	8,885
8	8,885	8,885
9	8,885	8,885
10	8,885	8,885
11	8,885	8,885
12	8,885	8,885
IRR =	12,0%	15,9%

Problem 14

Brew Corporation is considering two alternative bond issues. In option 1, Brew will make interest payments indefinitely at the rate of 5%. Under option 2, Brew will make the first interest payment in one year's time at 4%, but subsequent payments will increase at 3%. The discount rate is 6%. Which option should Brew choose? Assume that Brew borrows the same amount under each alternative.

Solution

Option 1 833,333 = 5% * 1000 / 6%

Option 2 1333,33 = 4% * 1000/ (6% - 3%)

Option 1 is better, since they are receiving $1,000 in exchange for promised payments worth only $833.