12. Break-even Point. Leverage. Financial Leverage. Combined Leverage

12.1 The break-even point

The break-even point or break-even quantity is that level of sales at which total revenues are exactly equal to total operating costs.

Operating costs are divided into three categories: fixed, variable and semifixed or semivariable.

Fixed operating costs - such as depreciation and insurance do not vary with level of production. Fixed operating costs are those costs that do not depend on the number of units produced within a given range of production (given plant capacity).

Variable operating costs are those expenses that vary directly with the level of production and sales.

The **break-even point** is given by:

$$q^* = \frac{F}{p - v} \tag{75}$$

where

q* is the break-even quantity,
F is fixed costs
p is price
v is unit variable costs

p-v in the denominator, called the contribution margin per unit, denotes the dollar amount that each unit sold „contributes" to meeting the fixed costs.

The mix of fixed and variable costs depends on the firm's choice of technology. It is clear, that a firm that has high fixed costs should generate more revenues in order to break-even. Therefore, more capital-intensive companies must produce and sell more just to survive.

In practice it is reasonable to continue an unprofitable project in the short run as long as its contribution margin is positive. If its variable costs are covered, the firm benefits from selling the product. The firm is not covering all costs (fixed and variable) and thus cannot guarantee profits. However, as long as the contribution margin is positive, the firm is covering all its variable costs and some of its fixed costs, thus reducing its losses. If the contribution margin is positive, the firm's short-term losses are reduced.

Assuming that the current sales is equal to 100% of capacity, the current break-even in per cent can be calculated using the following formula:

$$q^* = \frac{F}{(p-v)} = \frac{Fpq}{(p-v)pq} = \frac{FS}{(pq-vq)p} = \frac{FS}{(S-V)p} = \frac{Fq}{(S-V)} \tag{76}$$

12.2 Leverage

Operating Leverage

The **degree of operating leverage** (DOL) measures the percentage change in NOI for a given percentage change in sales:

$$\text{DOL} = \frac{\text{percentage change in NOI}}{\text{percentage change in sales}} \tag{77}$$

DOL is calculated in the following way:

$$\text{DOL} = \frac{q \times (p - v)}{q \times (p - v) - F} \tag{78}$$

$$\text{DOL} = \frac{S - V}{S - V - F} \tag{79}$$

DOL measures the sensitivity of NOI to changes in the firm's revenues.

Financial Leverage

Financial leverage measures the sensitivity of the firm's net income (NI) to changes in its net operating income (NOI). In contrast to operating leverage, which is determined by the firm's choice of technology (fixed and variable costs), financial leverage is determined by the firm's financing choices (the mix of debt and equity).

The degree of financial leverage (DFL) measures the percentage change in net income for a given percentage change in NOI:

$$\text{DFL} = \frac{\text{percentage change in NI}}{\text{percentage change in NOI}} \tag{80}$$

DFL is calculated in the following way:

$$\text{DFL} = \frac{q \times (p - v) - F}{q \times (p - v) - F - I} \tag{81}$$

$$\text{DFL} = \frac{S - V - F}{S - V - F - I} \tag{82}$$

where I is the interest expenses.

Combined Leverage

Combined leverage measures the overall sensitivity of the firm's net income (NI) to a change in sales. The **degree of combined leverage** (DCL) measures the percentage change in net income for a given percentage change in sales:

$$\text{DCL} = \frac{\text{percentage change in NI}}{\text{percentage change in net sales}} \tag{83}$$

DCL is calculated in the following way:

(84) $$DCL = \frac{q\,x\,(p-v)}{q\,x\,(p-v)-F-I}$$

(85) $$DCL = \frac{S-V}{S-V-F-I}$$

Example 7. Leverage Ratios

The Magic Co. has sales this year of $1000, variable costs account for
10% of revenues. It has fixed costs of $600, interests expense of $100
and a tax rate of 40%. The company currently has 100 shares outstanding.
The expected growth rate for revenues is 20%.
(a) What is the operating income, profit before tax, earnings, EPS, DOL,
DFL and DCL for alternative plans:
1. the company is unlevered and does not pay interests,
2. the company is levered.
(b) Calculate the expected operating income, net income and EPS
using DOL, DFL and DCL.
(c) Present the income statement for both plans.

Solution

(a)

		Plan 1	Plan 2
	Sales	1 000	1 000
	- Variable costs	100	100
	Contribution margin	900	900
DOL	- Fixed costs	600	600
DCL	Operating income	300	300
DFL	- Interest	0	100
	Profit before tax	300	200
	- Tax	120	80
	Earnings	180	120
	EPS	1.8	1.2

	Plan 1	Plan 2
DOL	3.00	3.00
DFL	1.00	1.50
DCL	3.00	4.50

(b)

Change in operating income	60%	60%
Change in earnings	60%	90%

(c)

	Plan 1			Plan 2		
	t=1	t=2	growth	t=1	t=2	growth
Sales	1 000	1 200	20%	1 000	1 200	20%
- Variable costs	100	120	20%	100	120	20%
Contribution margin	900	1 080	20%	900	1 080	20%
DOL - Fixed costs	600	600	0%	600	600	0%
DCL Operating income	300	480	60%	300	480	60%
DFL - Interest	0	0		100	100	
Profit before tax	300	480	60%	200	380	90%
- Tax	120	192	60%	80	152	90%
Earnings	180	288	60%	120	228	90%
EPS	1.8	2.9	60%	1.2	2.3	90%

Relationship between Book ROA and Book ROE

The return on assets (ROA) measures the accounting performance of the investment without regard to the manner in which the asset is financed. The return on equity (ROE) measures the net effects of both the investment and financing decisions.

(86) $$\text{Book ROA} = \frac{\text{NOI x (1 - T)}}{\text{Net Assets}}$$

(87) $$\text{Book ROE} = \frac{\text{NI}}{\text{Equity}}$$

The relationship between ROA and ROE is shown in equation:

$$(88) \quad ROE = ROA + \left[ROA - \frac{I}{D}(1-T) \right] \frac{D}{E}$$

Changes in ROA and ROE are related to DOL and DFL.

$$(89) \quad \%\Delta ROA = DOL \times \%\Delta S$$
$$(90) \quad \%\Delta ROE = DFL \times \%\Delta ROA$$
$$(91) \quad \%\Delta ROE = DCL \times \%\Delta S$$

Break-even, Leverage and Cash Flows

The cash flow break even point is calculated as:

$$(92) \quad BE_{CF} = \frac{F - U - \frac{TU}{(1-T)}}{P - V}$$

where

F is fixed costs
U is depreciation
F-U is cash fixed costs (fixed costs minus depreciation)
T is income tax rate
$\frac{TU}{(1-T)}$ is adjustment factor to convert accounting based (profits) analysis to cash flow based analysis.

The company's operating cash flows and net cash flows can be calculated as

$$(93) \quad OCF = [Q \times (P - V) - F](1 - T) + U$$
$$(94) \quad NCF = [Q \times (P - V) - F - I](1 - T) + U$$

The **degree of operating cash flow leverage,** DOL_{CF}, is the sensitivity of operating cash flows (OCF) to changes in sales. It can be calculated as:

$$(95) \quad DOL_{CF} = \frac{Q \times (P - V)}{Q \times (P - V) - (F - U) + \frac{TU}{(1-T)}}$$

The **degree of financial cash flow leverage,** DFL_{CF} measures the sensitivity of net cash flows (NCF) to changes in operating cash flows (OCF). It can be calculated using the following equation:

$$(96) \quad DFL_{CF} = \frac{Q \times (P - V) - (F - U) + \frac{TU}{(1-T)}}{Q \times (P - V) - (F - U) - I + \frac{TU}{(1-T)}}$$

The **degree of combined cash flow leverage,** DCL_{CF}, measures the sensitivity of net cash flows to changes in sales and is calculated as

$$(97) \qquad DCL_{CF} = \frac{Q\,x\,(P-V)}{Q\,x\,(P-V)-F-U-I+\frac{TU}{(1-T)}}$$

Task 12

1. Calculate Break-even Point
2. Calculate DOL, DFL and DCL

Problem 29. Break-even Point

Required:

Calculate and interpret break-even point.

Solution

$$q^{*} = \frac{F}{(p-v)} = \frac{Fpq}{(p-v)pq} = \frac{FS}{(pq-vq)p} = \frac{FS}{(S-V)p} = \frac{Fq}{(S-V)}$$

	Dec. 31, 2014	*Dec. 31, 2013*	*Dec. 31, 2012*
Fixed Costs = Selling, general and administrative expenses	17 218	17 310	17 738
Variable Costs = Selling, general and administrative expenses	17 889	18 421	19 053
Sales = Net Operating Revenues	45 998	46 854	48 017
Interest Payments	483	463	397
q	100%	100%	100%
Break-even Point	61,3%	60,9%	61,2%

Problem 30. Leverage

Required:

Calculate and interpret DOL, DFL and DCL

Solution

	Dec. 31, 2014	*Dec. 31, 2013*	*Dec. 31, 2012*
$DOL = \frac{S-V}{S-V-F}$	2,58	2,56	2,58
$DFL = \frac{S-V-F}{S-V-F-I}$	1,05	1,04	1,04
$DCL = \frac{S-V}{S-V-F-I}$	2,70	2,67	2,67
	2,70	2,67	2,67